Uncertainty Sources, Types and Quantification Models for Risk Studies

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Workshop on Risk Assessment and Safety Decision Making Under Uncertainty
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Outline

• Risk quantification and management
• Uncertainty in prediction and validation
• Knowledge and ignorance terminology
• Formalized languages
  – Generalized information theory
  – Generalized theory of uncertainty
• Uncertainty at the system level
• Open questions
**Risk Terminology**

**Risk**: The potential for loss or harm to systems due to the likelihood of an unwanted event and its adverse consequences.

Risk is an aggregate of (Hazard and scenarios, Consequences, Vulnerability, Threat rate)

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**Risk Assessment and Management**

1. What could happen?
2. How likely is it to happen?
3. What are the consequences if it happens?
4. What can be done?
5. What are the costs and benefits?
6. What effect will these actions have on future options?
Need for REAL Data

Data Sources for Quantitative Risk Analysis

1. Actuarial Data
   - Real data from experience with identical items in an identical environment & application

2. Published Data
   - Real data published on similar items in an identical environment & application

3. Engineering Judgment
   - Subjective estimates using expert opinion elicitation

4. Stress Modification
   - Data modification to reflect environmental and service stresses of intended application

5. Experience Modification
   - Modification of estimates as data accumulates from field experience

6. Working Data
   - Data application and re-application

Collection of actuarial data as filed experience is gained

“And will you be taking part in our toxicology study tonight?”
Risk Management

- Identify alternative risk mitigation strategies
- Assess benefits and costs of each
- Assess impact of strategy on future options

**Benefit = (Risk Before) – (Risk After)**

\[ \text{Benefit} = (\text{Risk Before}) - (\text{Risk After}) \]

**B/C Ratio = \frac{\text{Benefit}}{\text{Cost}}**

Hurricane Katrina: Risk Methodology
Hurricanes

Hurricane Katrina: Methodology

Start:
Define Project Objectives

Hazard Identification & Analysis

Definition of Hurricane Protection System

Failure Modes and Fragilities

Vulnerability Analysis & Assessment

Systems Analysis and Modeling

Water Volume Computations, Storage, and Interflow

Consequence Analysis: Economic and Life Loss

Elevation Exceedance

Uncertainty Analysis

Out of Study Scope

Decision Analysis: Benefits/Costs & Tradeoffs

Risk Informed Decisions
### Event Tree

**Hazard analysis** (hazard rates and effects)

- Hurricane (H) & rate (λ)
- Precipitation inflow (Q)
- Spatial peak surge & effective wave height (SWA) and durations

**Basin system probabilities & water volumes (conditional values per event)**

- Closure & operations (E)
- Overtopping (O)
- Drainage, pump & power (P)
- Net water levels (W)

**Basin consequences** (water volume, elevation & loss per event)

- Breadth (B)
- Evacuation effectiveness (E)
- Life loss (L)
- Economic loss (S)

<table>
<thead>
<tr>
<th>Hurricane (H) &amp; rate (λ)</th>
<th>Precipitation inflow (Q)</th>
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<tbody>
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<td>Spatial peak surge &amp; effective wave height (SWA) and durations</td>
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</table>

**Simulated Hurricane Tracks**

*Includes all failure modes of all reaches and their features*

\[
\lambda(C > c) = \sum_i \sum_j \lambda(P(h_j)P(S_j | h_j) \times P(C > c | h_j, S_j) \lambda)
\]
Hazard/Elevation Profiles

\[ \lambda(E > e) = \sum \lambda(P(h)P(S | h)P(E > e | h, S)) \]

All storms & branches

Results provided are for illustration purposes.
Rail Safety and Security

4 Different Influence Circles

An Example Layer Representing Vulnerability

Zone to Protect along the railroad

Inventory: people, structures, schools, utilities, police resources, fire stations, hospitals, …

Requirements for risk methods

• A multi-hazard quantitative risk framework for informing decisions:
  – Analytic
  – Quantitative
  – Probabilistic
  – Consistent
  – Transparent
  – Defensible

• Reliability of knowledge
Verification, Validation and Accreditation

Model Validation

Validation and application domains (Sandia report)
Model Validation

Validation methods (Sandia report)

(a) Graph Norm

(b) Deterministic

(c) Experimental Uncertainty

(d) Numerical Error

(e) Nondeterministic Computation

(f) Quantitative Comparison

Validation: Capsize Risk

Hypothesis testing

Input Hypothesis testing

Output Hypothesis testing

Model tests

Simulation runs (FREDYN or TEMPEST)

Wave spectra

Response spectra
Definition of Validation

- **Validation**
  The process of determining the degree to which a model is an accurate representation of the real-world from the perspective of the intended uses of the model.

Knowledge & Ignorance

- **Notions, representations and measures**
  - Knowledge and ignorance
  - Information and uncertainty
  - Other considerations
    - Opinion
    - Language
    - Cognitive processes
Knowledge & Ignorance

• The greatest enemy of knowledge is not Ignorance, it is the Illusion of knowledge

Stephen Hawking

Knowledge & Ignorance

• Knowledge can be defined as justified true beliefs (JTBs)
• Knowledge is subjective or relative, and cannot be separated from the human experience (model-dependent reality)
• Knowledge can be fallible
• Reliability of knowledge
• Evolutionary epistemology
Evolutionary Epistemology

Knowledge & Ignorance

- The object of reasoning is to find out, from the consideration of what we already know, something else which we do not know.
  
  C. S. Peirce

- It takes considerable knowledge to realize the extent of self ignorance.
  
  Thomas Sowell
Knowledge & Ignorance

- Compared to our pond of knowledge, our ignorance remains Atlantic
- Invited scientists to state what they would like to know in their respective fields, and noted that the more eminent they were the more readily and generously they described their ignorance

Duncan and Weston-Smith

Classification of Ignorance

- Evidential reasoning (random sets)
- Ignorance
- Open world (unforseen events)
- Conscious Ignorance
  - Inconsistency
  - Incompleteness
  - Confusion
  - Inaccuracy
  - Conflict
  - Approximations
  - Vagueness

- Blind Ignorance
  - Fallacy
  - Unknowable
  - Irrelevance
  - Unknowable
  - Untopicality
  - Undecidability
  - Taboo
  - Uncertainty
  - Absence
  - Randomness
  - Probability & statistics

- Incompleteness
- Unknowability
- Absences
- Randomness
Identification and Classification of Theories

<table>
<thead>
<tr>
<th>Universal Set</th>
<th>Elements of Universal Set</th>
<th>Set (or Event as a Notion)</th>
<th>Element Belonging to a Set</th>
<th>Comments Including an Example Applicable Theory</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Precise</td>
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<td>Crisp sets</td>
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<td>Imprecise</td>
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<td>Closed-World Assumption</td>
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<td>Fuzzy sets</td>
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Aleatory and Epistemic Uncertainties

*Inherit randomness (i.e., aleatory uncertainty)*
- It cannot be reduced or eliminated by enhancing the underlying knowledge base.
- Examples: wave loads on an offshore platform, strength properties of materials

*Subjective (or epistemic) uncertainty*
- Uncertainty is also present as a result of a lack of complete knowledge. It can be reduced as a result of enhancing the state of knowledge by expending resources
- Example: Consequences
Aleatory and Epistemic Uncertainties

Combined uncertainty

\[ P = \overline{P} \hat{P} \]

\[ \hat{P} = LN[1.0, COV(\hat{P})] \]

\[ COV(P) = \sqrt{[COV(\overline{P})]^2 + [COV(\hat{P})]^2} \]

Classifying Monotone Measures

Boolean Algebras:
Classical sets and propositions

Classical probability theory

Classical measures:
Additive

Generalizations

Weaker algebras:
Fuzzy sets or propositions of various types

Weaker measures:
Monotone with various special properties
Classifying Monotone Measures

- **Classical probability theory**: classical probability (additive) functions defined on classical (crisp) sets.
- **Probability theory based on fuzzy events**: classical probability (additive) functions defined on fuzzy sets.
- **Dempster-Shafer Theory (DST) of evidence**: a pair of special semicontinuous monotone measures, called belief and plausibility measures, which are defined on classical sets and which conveniently represent lower and upper probabilities, respectively.
- **Theory based on feasible interval-valued probability distributions (FIPD)**: according to the FIPD, lower and upper probabilities are determined for all sets $A \in PX$ by intervals of probabilities on singletons $(x \in X)$.

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Generalized Information Theory

- **Generalized Information Theory (G. Klir)**:
  - **Level 1**: Find an appropriate mathematical representation of the conceived type of uncertainty
  - **Level 2**: Develop a calculus by which this type of uncertainty attributes can be properly quantified and manipulated
  - **Level 3**: Find a meaningful way of measuring relevant uncertainty in any formalized in the theory
  - **Level 4**: Develop methodological aspects of the theory, including procedures for making the various uncertainty principles operational within the theory
Generalized Theory of Uncertainty — Lotfi A. Zadeh

- Uncertainty is an *attribute of information*
- Information is *conveyed by constraining* the values of a variable
- Proposition is a *carrier of information*
- Proposition = generalized constraint

- Example:
  Critical pressure is 500 ksi
  – constrains pressure

Closed-World Versus Open-World Assumption

- Mathematical definitions based on the universal ($\Omega$) and null ($\emptyset$) sets
  - Closed world
    $m(\emptyset) = 0$
    $Bel(\Omega) = 1$
  - Open world
    $m(\emptyset) \geq 0$
    $Bel(\Omega) \leq 1$
- Inconsistency based on a body of evidence
  – A high level of inconsistency $\rightarrow$ unseen events or nonempty “null set”
Closed-World Versus Open-World Assumption

- Patterns:
  - Computational linguistics, Cryptography

\[ S = C, C, P, C, B, B, P, C \]

where

\[ C = \text{cyber attack (1)} \]
\[ P = \text{perimeter breach (2)} \]
\[ B = \text{bomb attack (3)} \]

Pattern \( S = 11213321 \)

What is the probability of an unseen event (U)?
Open Questions

- A unified theory:
  - Knowledge and ignorance
  - Information and uncertainty
- Foundational bases:
  - Generalized Information Theory
  - Generalized Theory of Uncertainty
- Uncertainty types and quantification methods
- Open world and pattern analysis

Selected Publications

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