Quantitative Methods for Decision-Making Under Uncertainty

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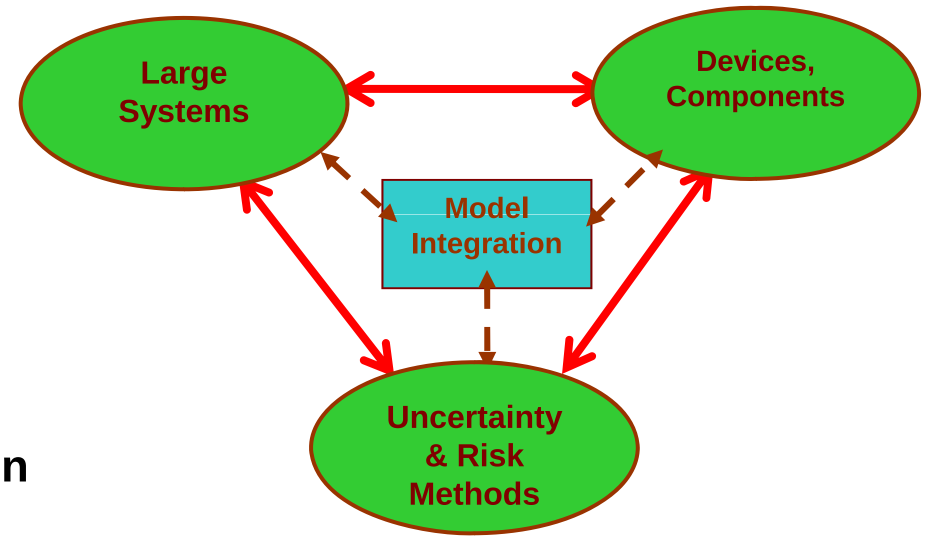
Reliability and Risk Engineering, Analysis, and Management

NSF-IGERT Graduate Program at Vanderbilt

Educational and Research Themes

- Multidisciplinary integration
- Large, complex systems
- Modeling and simulation
- Economic, legal, regulatory, and social perspectives



Cross-cutting methodologies

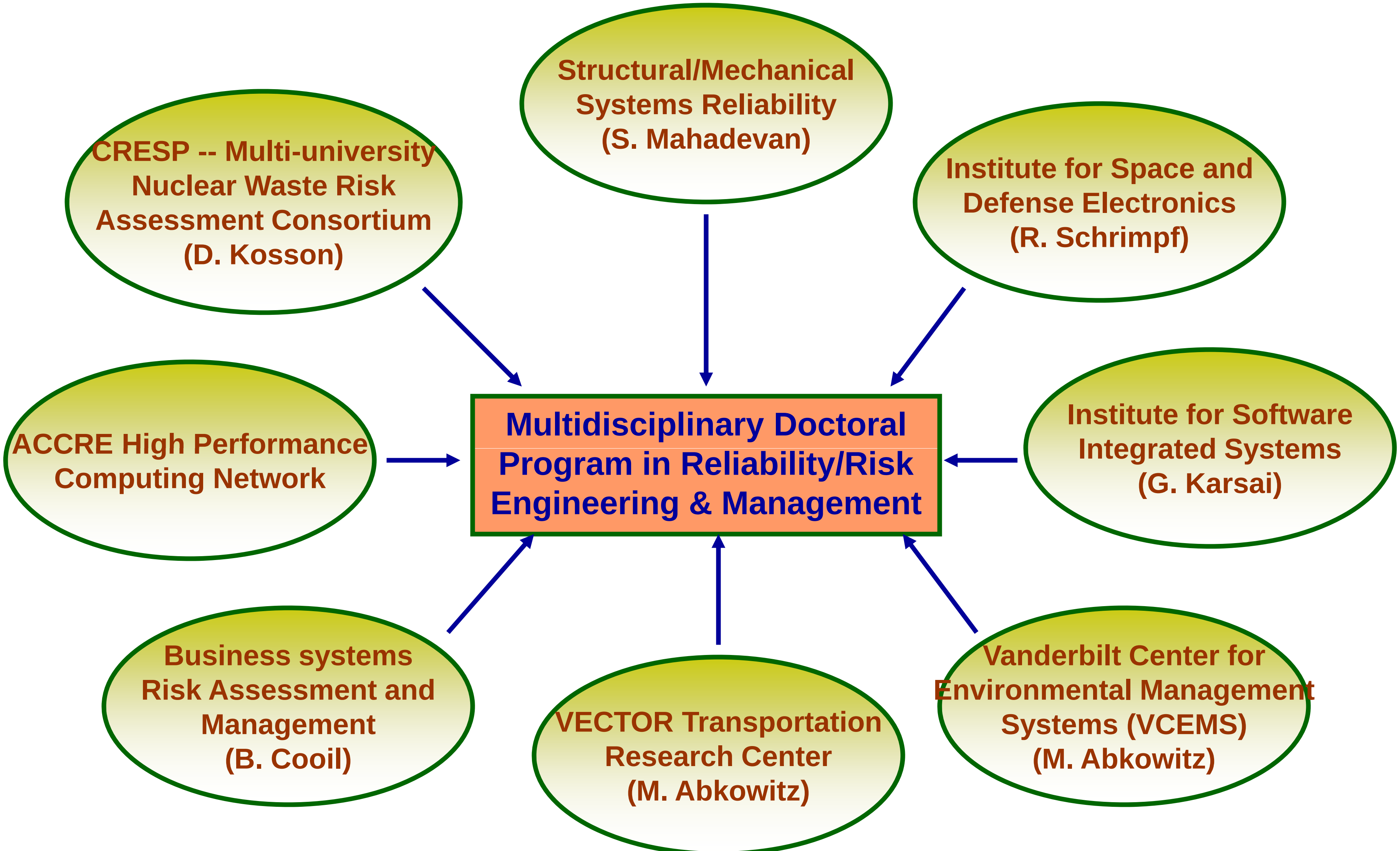
- Uncertainty quantification, propagation
- Risk quantification
- Decision-making under uncertainty

Participants

- 42 graduate students (34 Ph.D., 8 M.S.)
- 30 professors → Engineering, Math, Economics, Business, Psychology, Medicine
- Summer researchers (undergraduates, high school teachers)

Reliability and Risk Engineering, Analysis, and Management

Multiple Resources at Vanderbilt



Industry & Government Support

INDUSTRY

- Boeing
- Kellogg, Brown & Root
- Fedex
- General Motors
- Chrysler
- General Electric
- Pratt and Whitney
- Bell Helicopter
- Medtronic
- Union Pacific
- Xerox

Laboratories

- Southwest Research Institute
- Transportation Technology Center

GOVERNMENT

- U. S. DOD (AFRL, Army, Navy, AFOSR)
- U. S. DOE (EM)
- U. S. DOT (FHWA)
- NASA (LaRC, MSFC, GRC, ARC, JPL)
- DOE Labs (SNL, LANL, INL, SRNL)
- Nuclear Regulatory Commission
- Federal Aviation Administration

Nature of support
Summer internships
Collaborative research projects
Advisory committee membership
Seminar speakers
Student recruitment

Risk Analysis Issues

- System modeling
  - Physics-based behavior models → finite elements, bond graphs
  - Surrogate models (GP, PC, RBF, NN)
  - Fault trees, event trees, petri-nets
  - Bayes networks

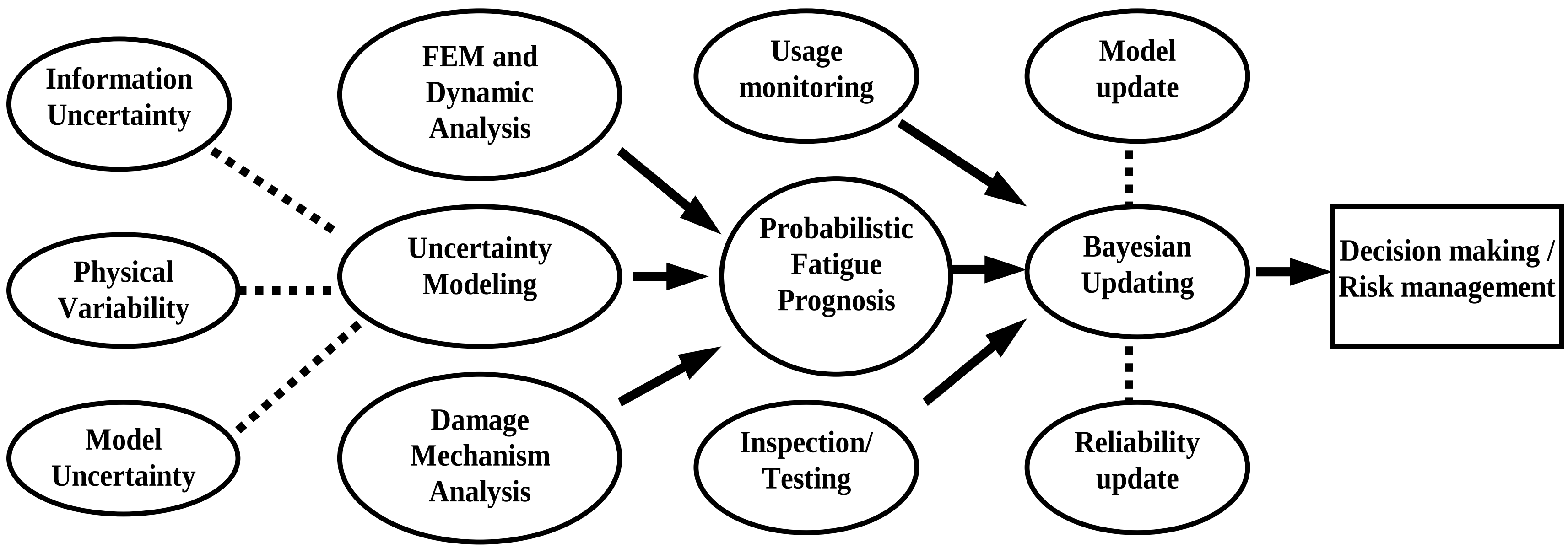
- Risk analysis
  - Multi-level -- Material → component → subsystem → system
  - Risk variation over space and time
  - Multi-physics, multi-scale problems

- Data Uncertainty
  - Sparse data, interval data, measurement uncertainty
  - Expert opinion
  - Heterogeneous information

- Model Uncertainty
  - Model form, model parameters
  - Errors → some deterministic, some stochastic

Reliability and Risk Engineering, Analysis, and Management

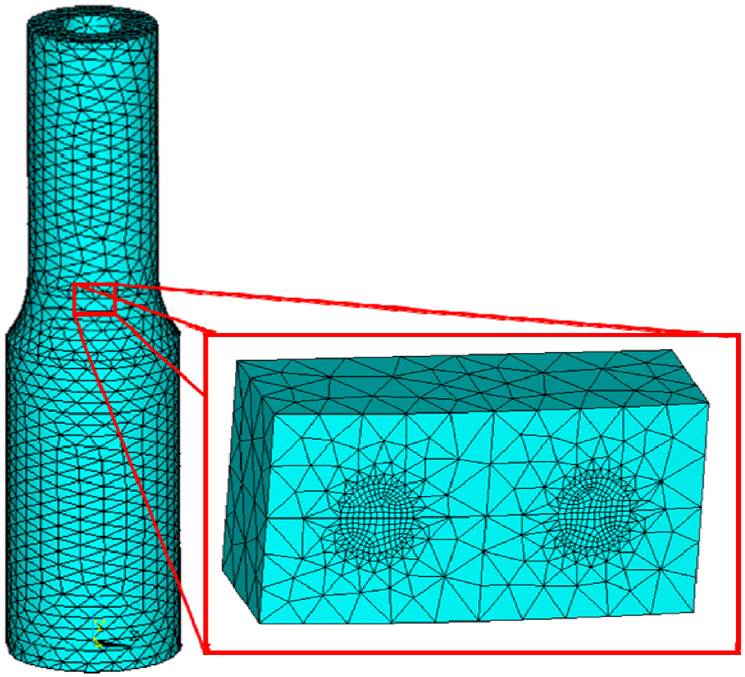
- Materials durability, fatigue, fracture
- Systems health diagnosis and prognosis
- Decision-making under uncertainty
- Model uncertainty, calibration, validation



Rotorcraft Damage Tolerance (FAA)

Rotorcraft mast

- Two-diameter hollow cylinder
- Elliptical surface crack in fillet region
- Sub modeling technique → Accuracy in stress intensity factor



Analyses

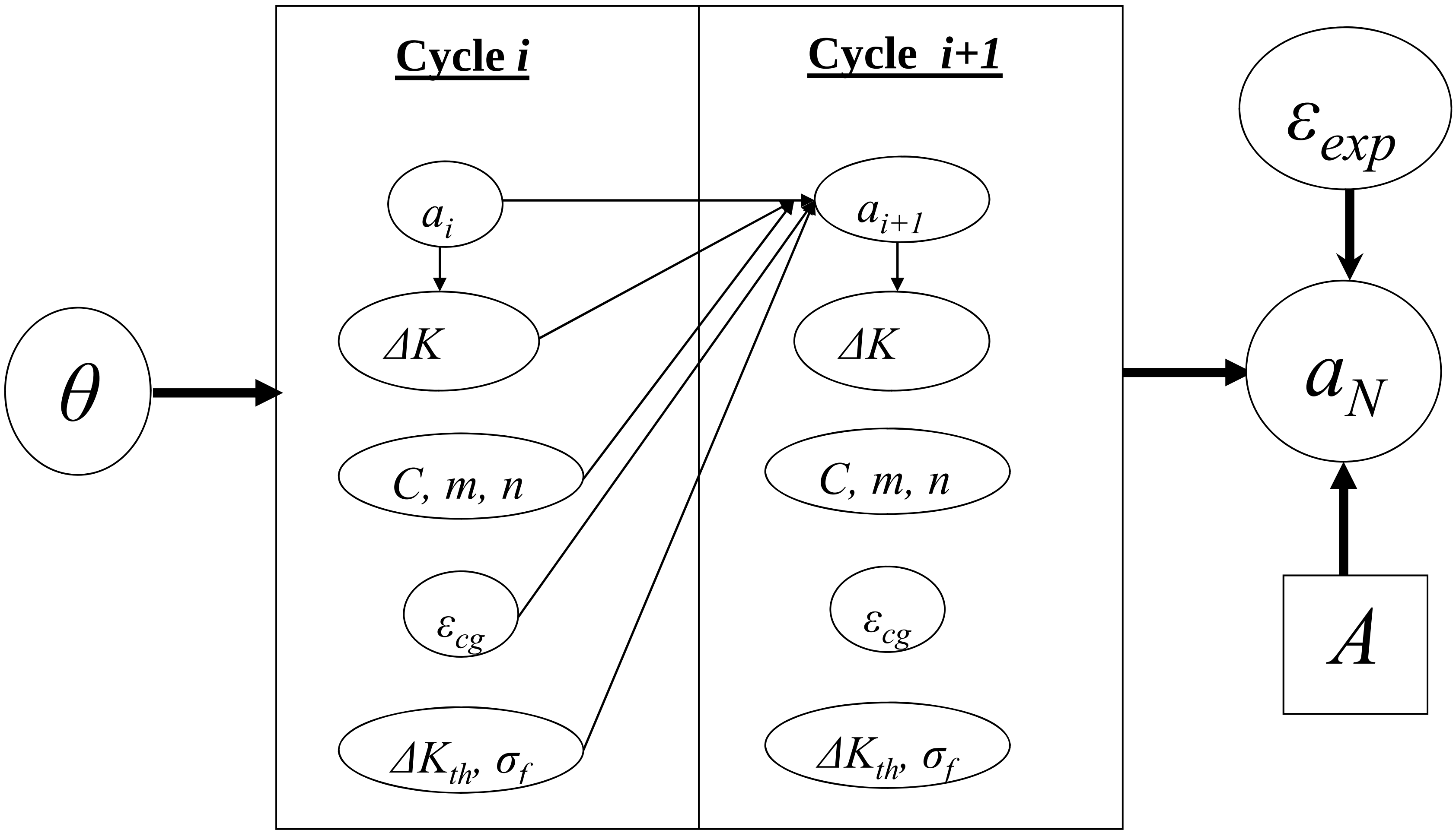
- Model calibration
  - Calibrate EIFS, model parameters
  - Estimate model errors in different stages of modeling
- Model validation
- Prediction uncertainty quantification
- Global sensitivity analysis
- Load monitoring and updating

Sources of Uncertainty

- Physical variability
  - Loading
  - Material Properties
- Data uncertainty
  - Sparseness of data used to quantify material properties
  - Output measurement uncertainty (final crack size, detection probability)
- Model uncertainty/errors
  - Analysis assumptions → LEFM, planar crack
  - Finite element discretization errors
  - Combination of multiple crack modes
  - Approximation due to surrogate model
  - Crack growth law → model form
  - Model parameters → crack growth model

    initial flaw size

Dynamic Bayes Network



Model Validation Metrics

Model response x
Observed values $\boldsymbol{y}$

Null Hypothesis $H_0: \boldsymbol{y} \in f(x)$

Alternative Hypothesis: $H_1: \boldsymbol{y} \notin f(x)$

Classical hypothesis testing

$H_0: E(\boldsymbol{y}) = E(x) \qquad Var(\boldsymbol{y}) = Var(x)$

$H_1: E(\boldsymbol{y}) \neq E(x) \qquad Var(\boldsymbol{y}) \neq Var(x)$

t test
chi-square test

Bayesian hypothesis testing

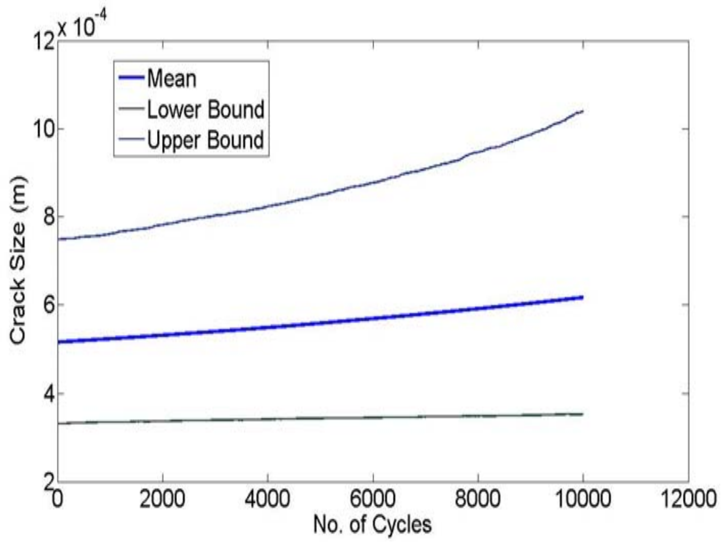
From Bayes theorem →

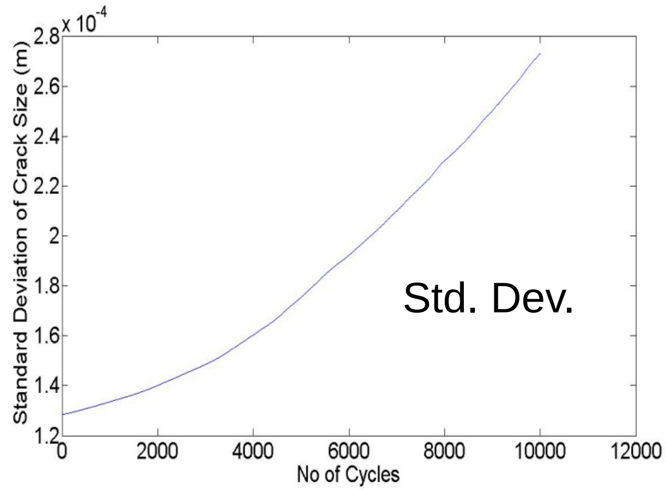
$$\frac{P(M_i|\text{observation})}{P(M_j|\text{observation})} = \left[\frac{P(\text{observation}|M_i)}{P(\text{observation}|M_j)}\right]\left[\frac{P(M_i)}{P(M_j)}\right]$$

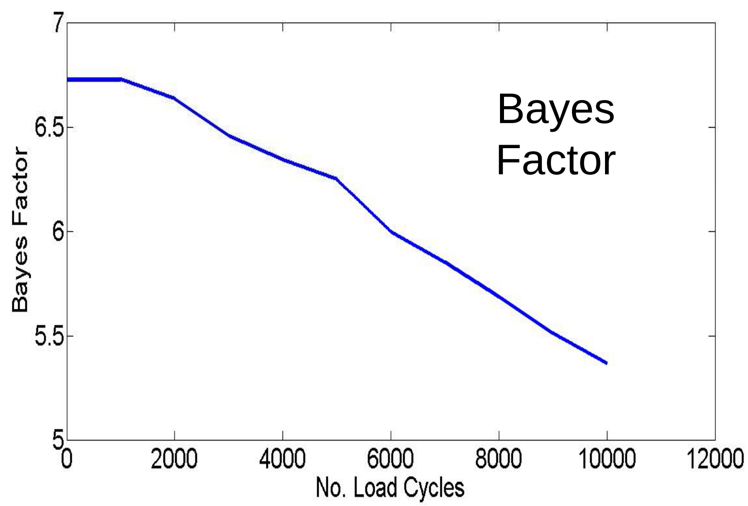
Bayes Factor
B

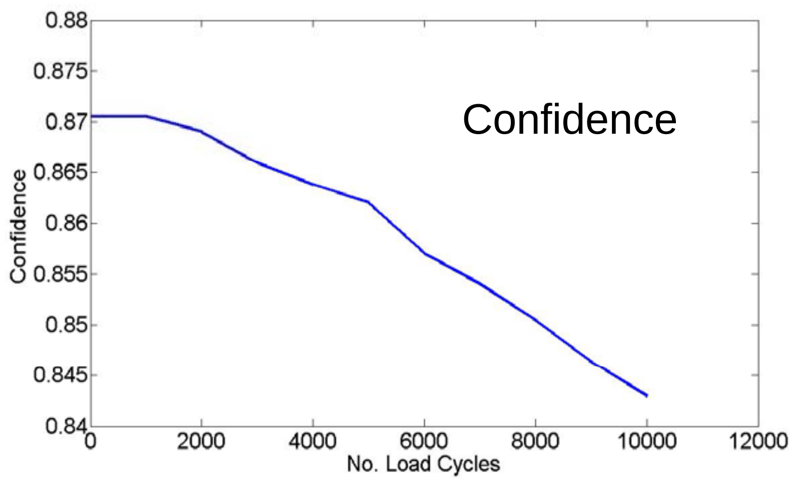
"Confidence"
= B/B+1

Crack size prediction UQ





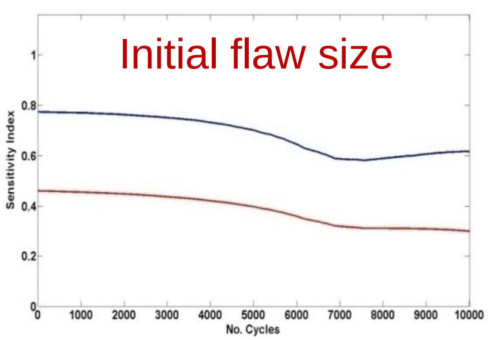


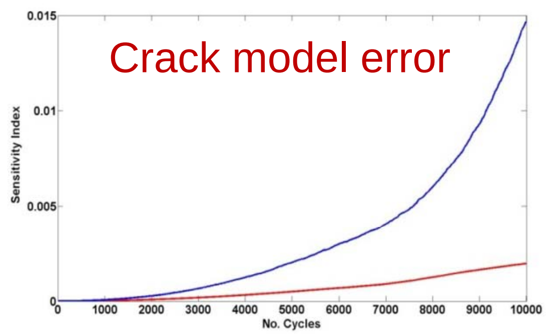


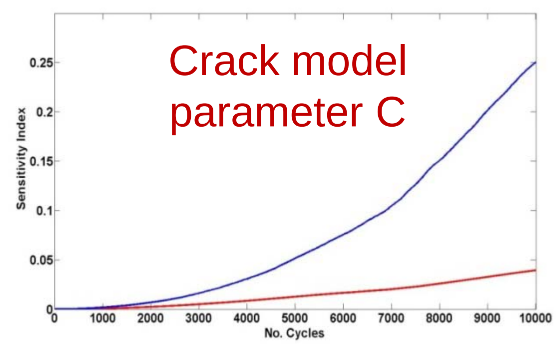
Sensitivity Analysis

- Local → Only one uncertainty is considered and all others are ‘frozen’ at the mean values
- Global → Analyze sensitivity of output over the entire domain of inputs rather than at mean values
- First order effects (S) & Total effects (S_T)

$$S_i = \frac{V_{X_i}(E_{X_{\sim i}}(Y \mid X_i))}{V(Y)} \qquad S_{T_i} = \frac{E_{X_{\sim i}}(V_{X_i}(Y \mid X_{\sim i}))}{V(Y)} = 1 - \frac{V_{X_{\sim i}}(E_{X_i}(Y \mid X_{\sim i}))}{V(Y)}$$





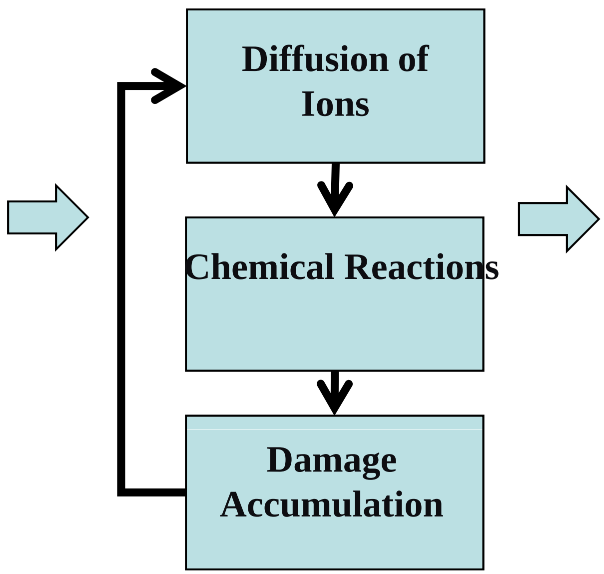


Cementitious barrier PA

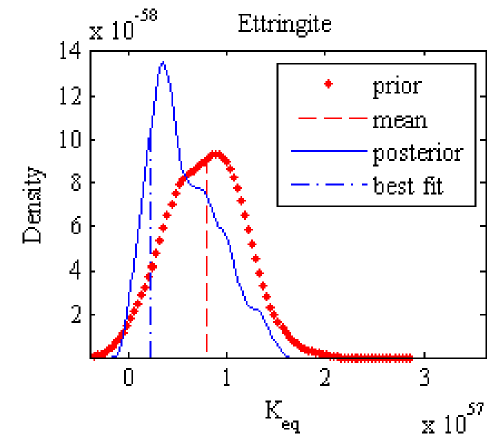
Multi-physics analysis

Inputs

- **Porosity**
- **Composition**
- **Modulus**
- **…..**

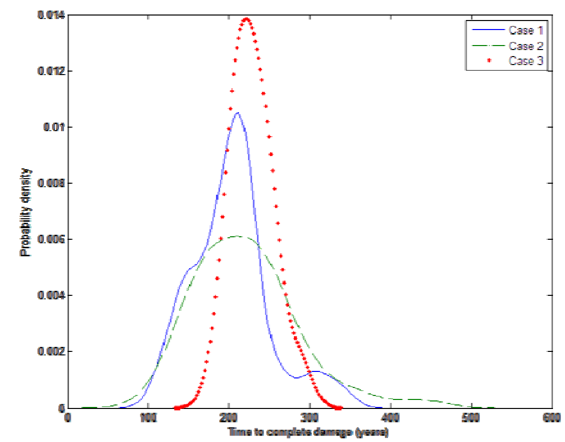


Damage Progression



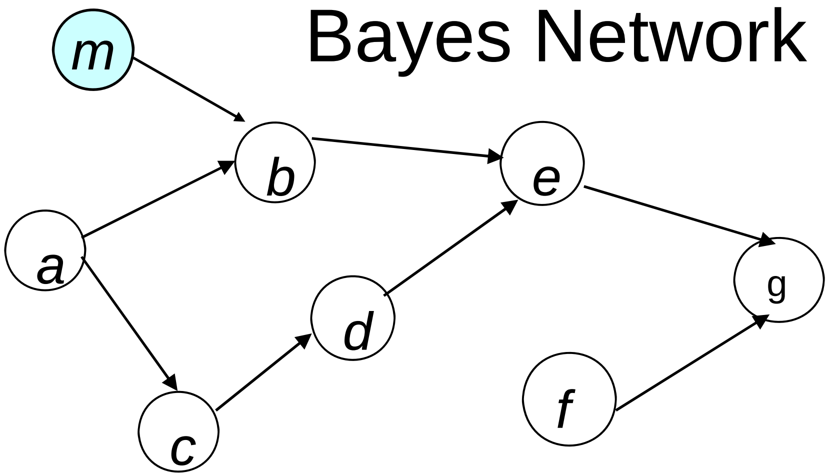
Calibration of chemical reaction model parameters (equilibrium constants)

Durability prediction



Extrapolation from Validation to Application

Bayes Network



Use for

- Calibration
- Validation
- Extrapolation

- **MCMC techniques**
- **Gibbs sampling**

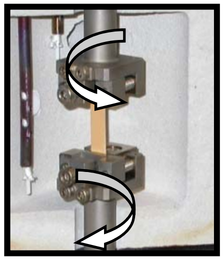
<u>Extrapolation scenarios</u>

- Nominal values to Extreme values
- Test conditions to Use conditions
- Validation variables to Decision variables
- Components to System

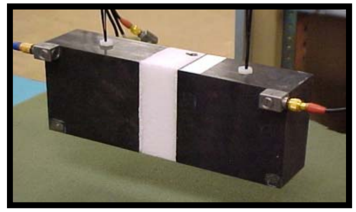
UQ in system-level prediction

Level 0 | **Level 1** | **Level 2** | **System level**

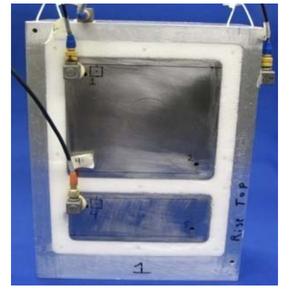
Foam



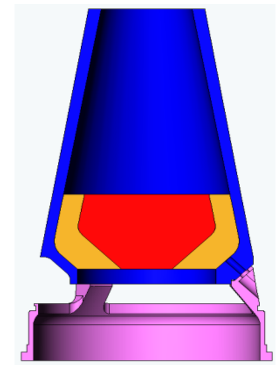
Material characterization



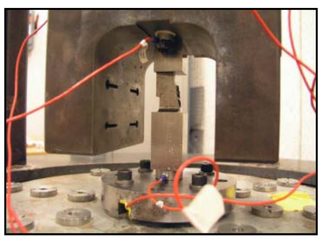
Component level

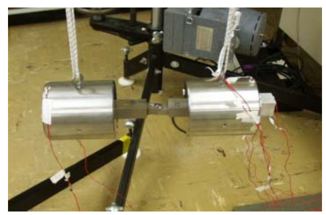


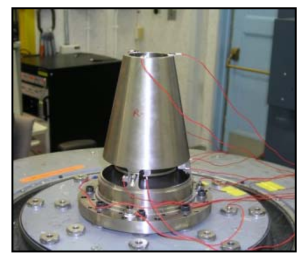
Sub-system level



Joints







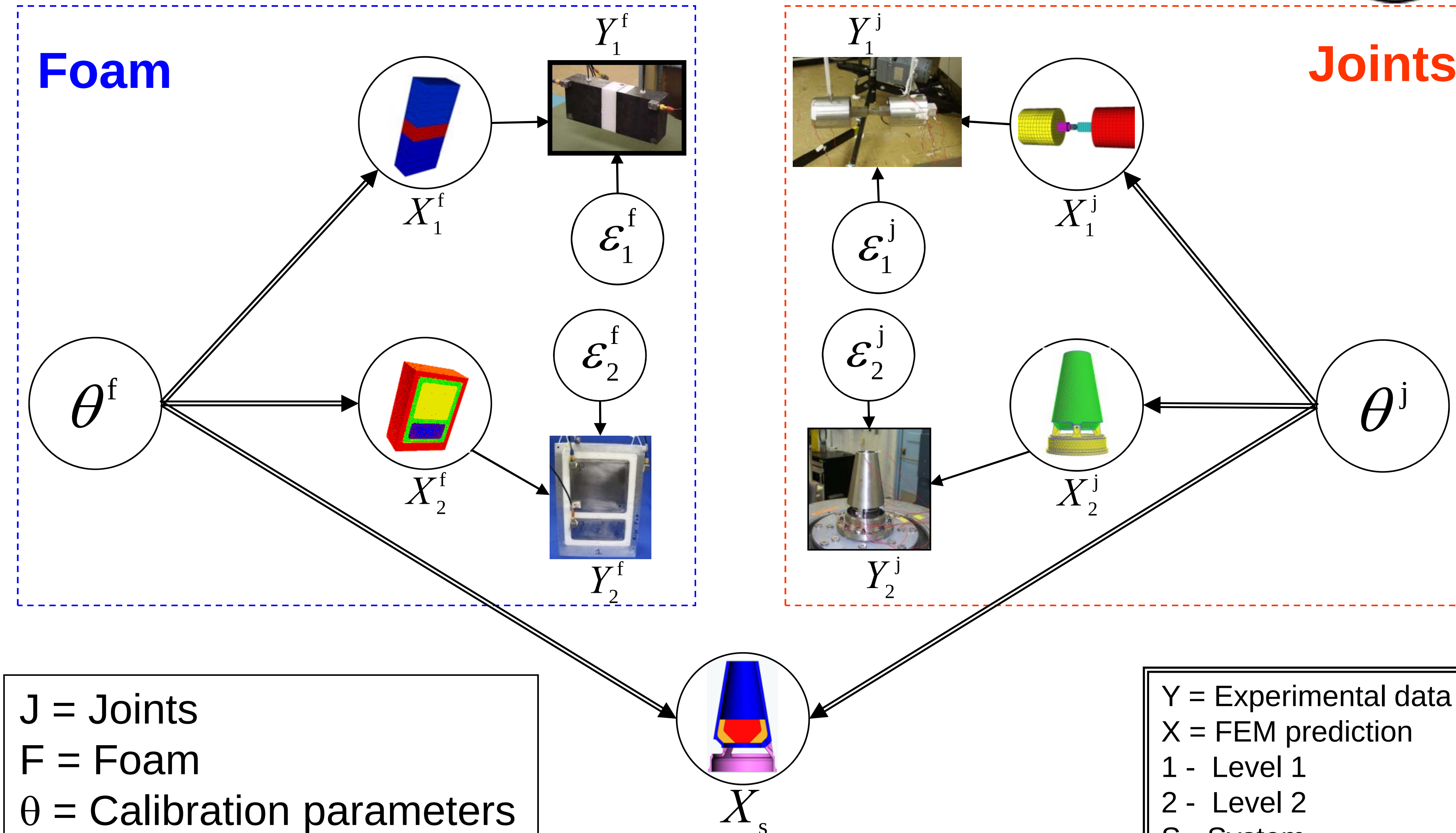
Hardware data and photos courtesy of Sandia National Laboratories

System complexity → Increases

Sources of uncertainty → Increase

Amount of real data → **Decreases**

Bayes Network Implementation



J = Joints
F = Foam
θ = Calibration parameters
ε = Error terms

Y = Experimental data
X = FEM prediction
1 - Level 1
2 - Level 2
S - System

Likelihood Approach to Data Uncertainty

- Likelihood function

$$L(P) \propto \left(\prod_{i=1}^{n} \int_{a_i}^{b_i} f_X(x \mid P)dx \right) \left(\prod_{i=1}^{m} f_X(x_i \mid P) \right)$$

interval data — sparse point data

P – distribution parameters
m – point data size
n – interval data size

- Maximum Likelihood Estimate → Maximize *L(P)*
- To account for uncertainty in *P* → $f(P) = \dfrac{L(P)}{\int L(P)}$
- Two approaches
  - Family of distributions for X (for every sample of P → probability distribution for X)
  - Single distribution of X

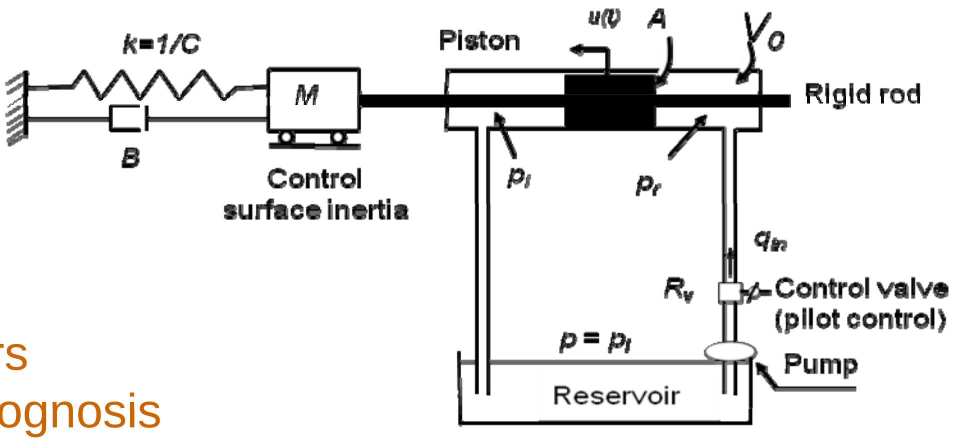
$$f(x) = \int f(x \mid P) f(P) dP$$

- Can use non-parametric distributions

Risk Management: System Health Monitoring

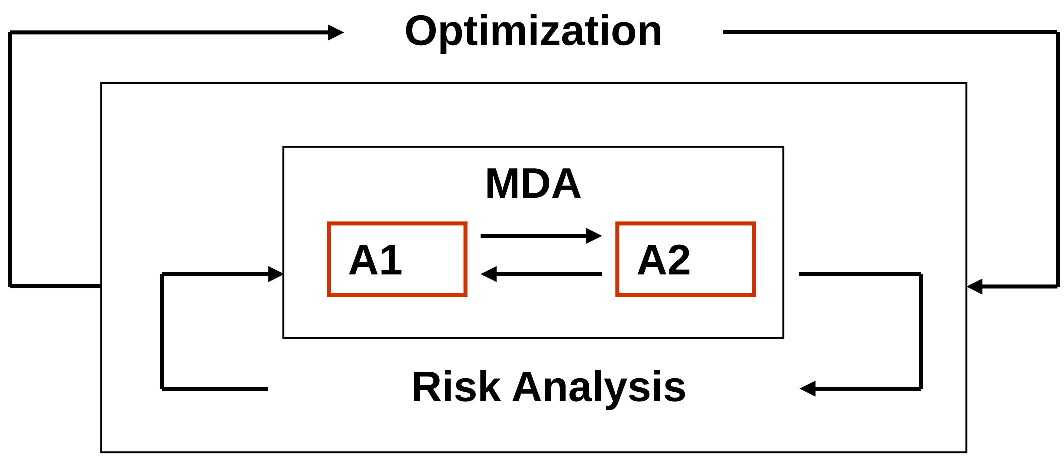
- System integration
  - Integrate reliability/risk methods with SHM
  - Integrate diagnosis with prognosis

- Rapid diagnosis and prognosis
  - Derive damage signatures
  - Qualitative isolation, then damage quantification



- Uncertainty Quantification
  - Quantify variability, uncertainty, errors
  - Estimate Confidence in diagnosis/prognosis

Decision-Making Under Uncertainty



- Various stages in life cycle → design, operations, maintenance
  - Multiple objectives, MCDA, decision trees, utility-based formulations
  - Multi-disciplinary systems
  - Optimization for reliability and robustness
  - Include both aleatory and epistemic uncertainties

- Dynamic, network systems
  - Critical facility protection – design of safeguards/detectors
  - Transportation networks, supply networks, emergency response systems

- System of systems
  - Multiple system linkages
  - Homeland security, military, commercial applications

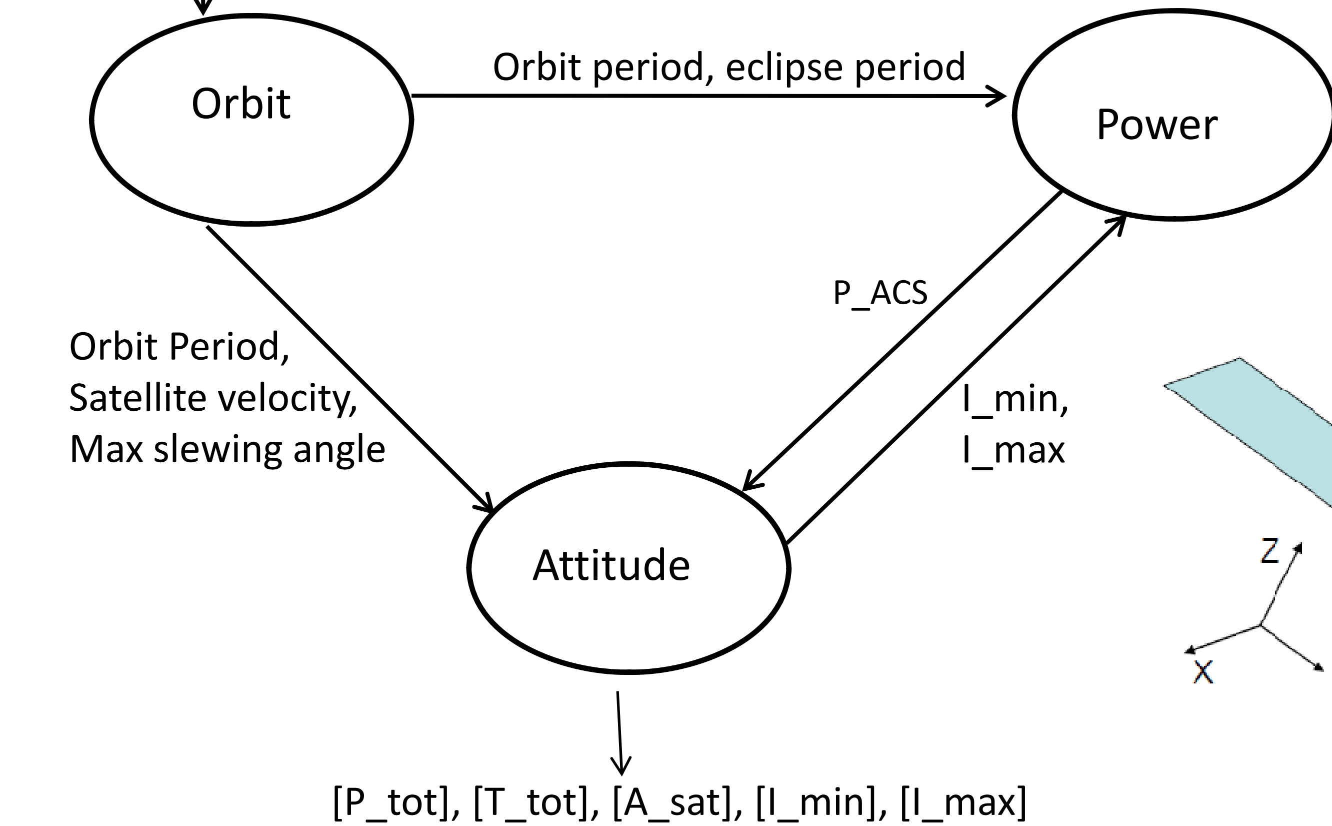
Fire Satellite System

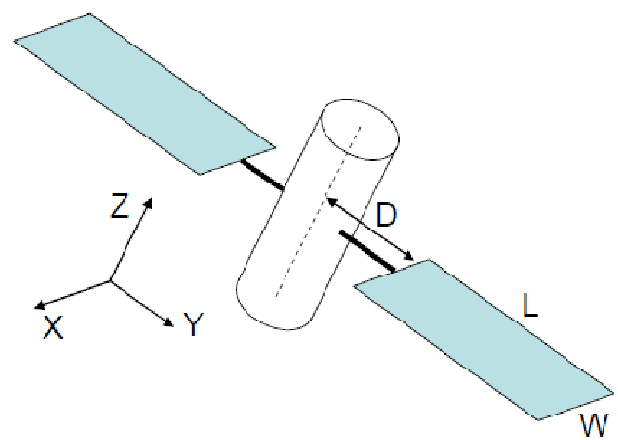
Target latitude,
Target longitude
Target size
[Altitude]

<u>Analyses</u>

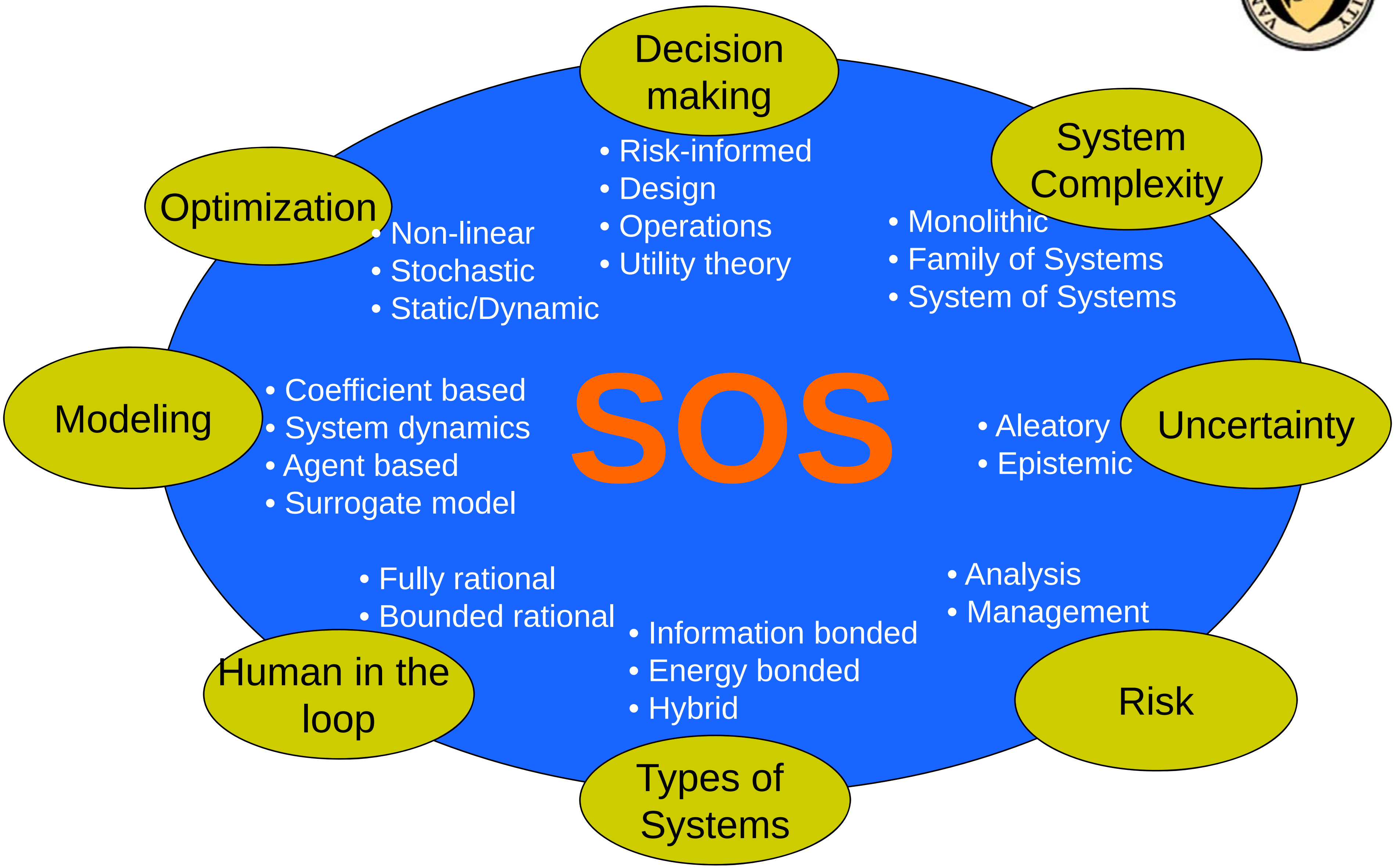
Multi-disciplinary uncertainty propagation

Design optimization for reliability, robustness



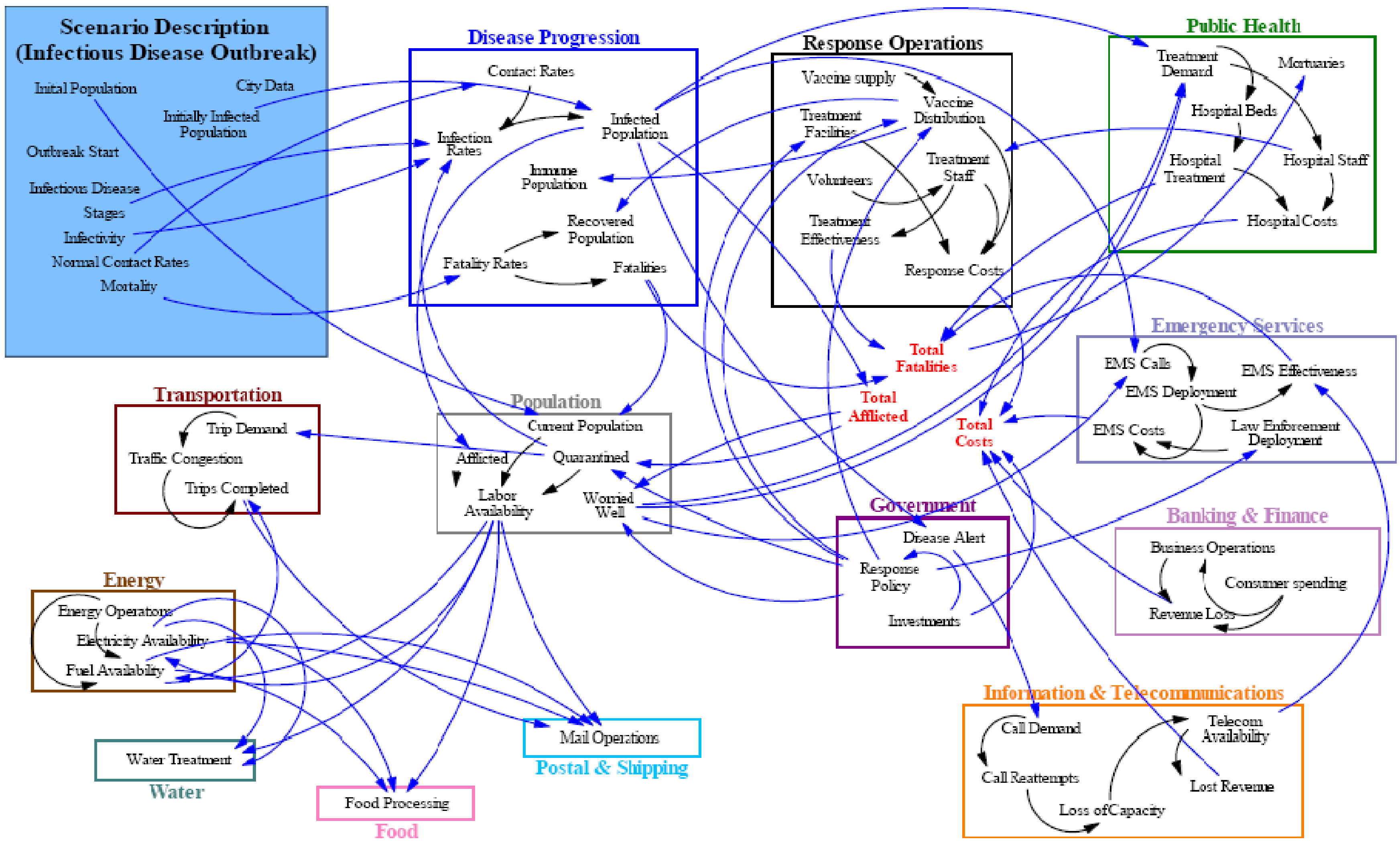


System of Systems Decision-Making Under Uncertainty



Pandemic Influenza Risk Management

CIPDSS (LANL)



Conclusion

Continuing opportunities for methods development

- System risk assessment
  - Risk variation with time and space
  - Dynamic, multi-physics, systems of systems
  - Computational effort
- Decision-making under uncertainty
  - Design, operations, maintenance, risk management
  - Data collection, Model development
  - Embedding flexibility
- Include data uncertainty
  - Sparse, noisy, qualitative, missing data, intervals, expert opinion
  - Multi-scale fusion of heterogeneous information
- Include model uncertainty
  - Validation, calibration, error estimation, extrapolation