Reactors and Fuels

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Major Topics

- Neutron interactions with matter and reactor control
- Nuclear reactors
- Nuclear fuels: fresh and spent

Interaction of Neutrons with Matter and Reactor Control

Interactions of Neutrons with Matter

Microscopic Neutron Cross Section

- Probability that a neutron will interact with a nucleus
- Measured in cm^2, denoted by σ
- Cross sections are on the order of 10^{-24} cm^2 with a typical range of ±100x
- To early researchers this was unexpectedly large - as big as a barn – and so it is called
- The cross section is not a measurement of the physical cross section of the nucleus
  - More a measure of how much more stable the nucleus will be after the interaction occurs

Reaction Rate

- Macroscopic cross section: Microscopic cross section (cm^2) multiplied by density of target nuclide (N, atoms/cm^3)
  – Denoted by Σ
- When multiplied by the neutron flux (φ, neutrons/cm^2-sec) the result is the interaction rate (R, interactions/cm^3-sec)
- $R = N*\sigma* \varphi = \Sigma* \varphi$

Neutron Flux

- The neutron flux is the total path length covered by all neutrons going in all directions in a cubic centimeter

$\varphi = n*v$

where

n = neutron density, n/cm^3-sec

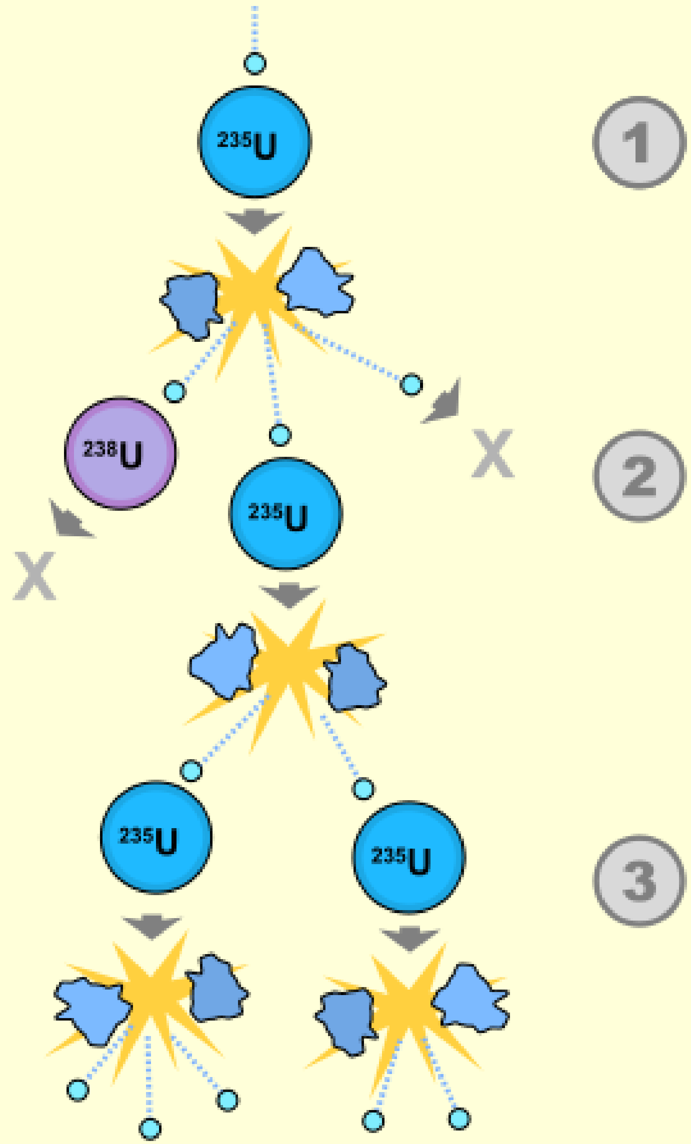
v = neutron velocity, cm/sec

Nuclear Material Definitions

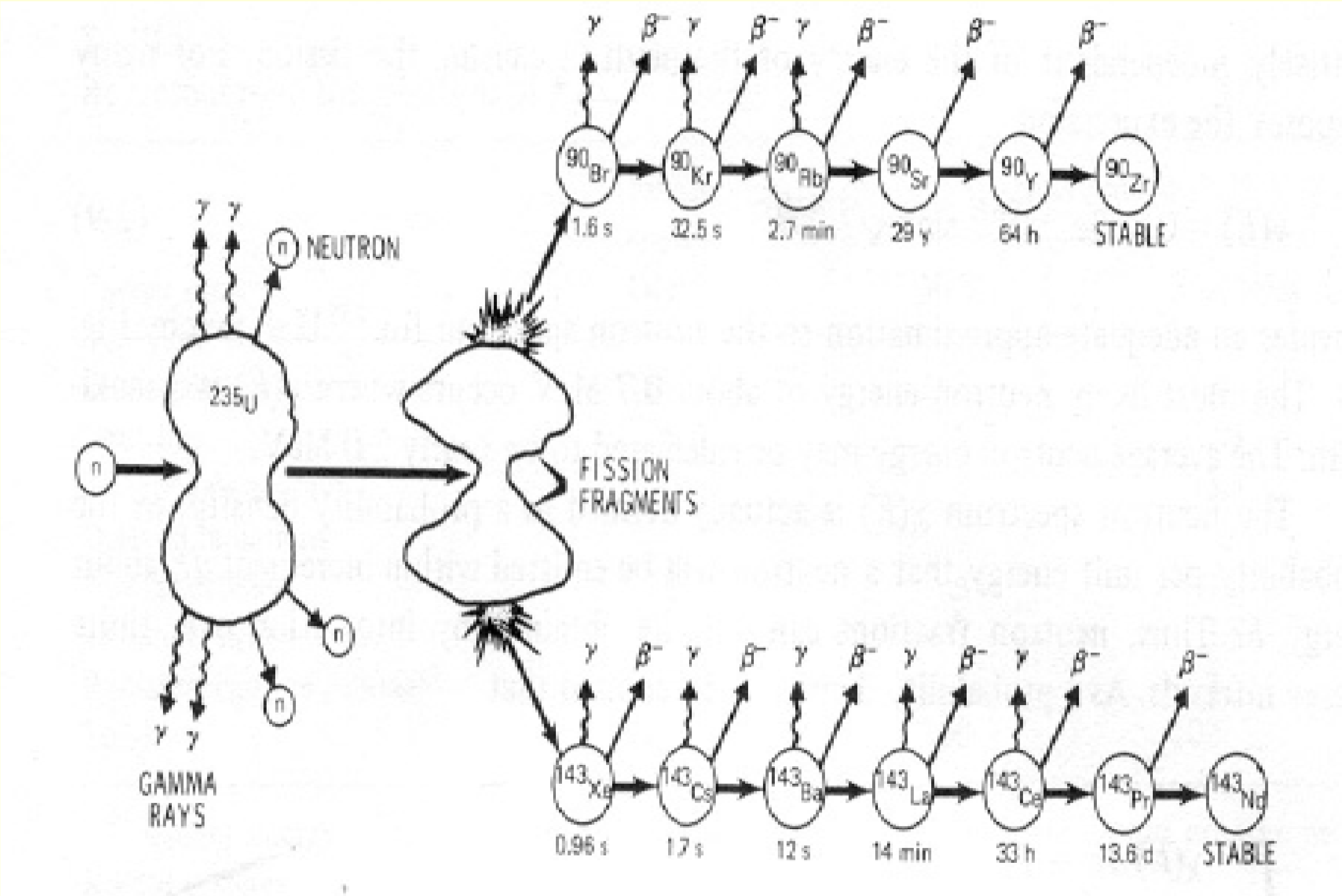
- Fissile: A nuclide that can support a self-sustaining nuclear reaction
  - ^{233}U, ^{235}U, ^{239}Pu, ^{241}Pu
  - Others which can only be made in small amounts
  - Only ^{235}U occurs naturally
- Fissionable/fissible: Nuclide can fission but not support a self-sustaining nuclear reaction
  - Virtually any actinide given high-energy neutrons
- Fertile: A nuclide that can be converted into fissile material
  - ^{238}U, ^{240}Pu, ^{232}Th

Chain Reaction

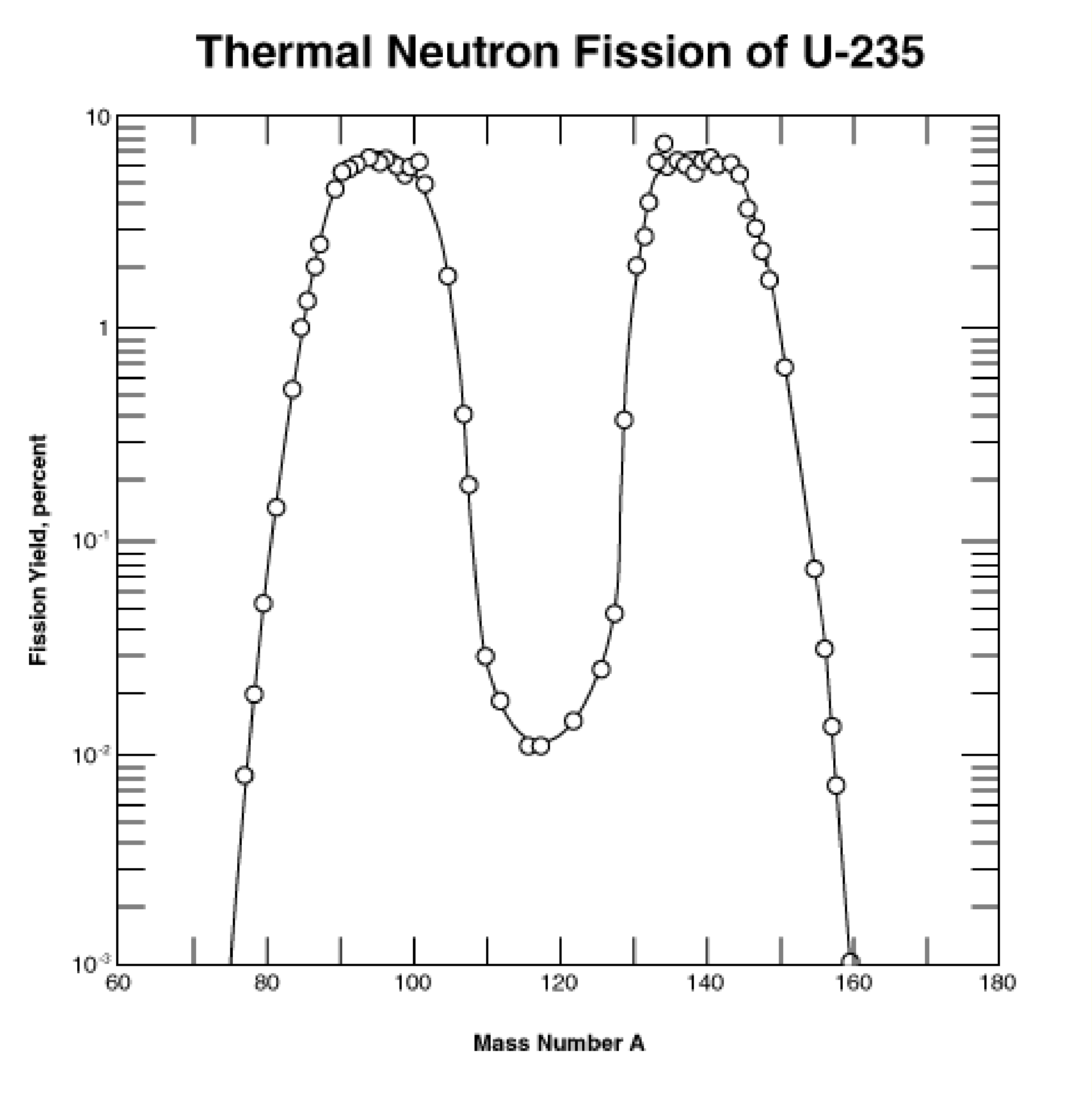
- Self-sustaining nuclear reaction



Fission Product Decay Chain-1



Fission Product Yield Curve



Binding Energy

- The mass of a nucleus is less than the sum of the masses of its protons and neutrons
- The difference is the energy that holds the nucleus together: binding energy
- Fission produces fission products and neutrons having less total binding energy
- The difference is the energy released in fission
  - Nuclear binding energy = Δmc^2

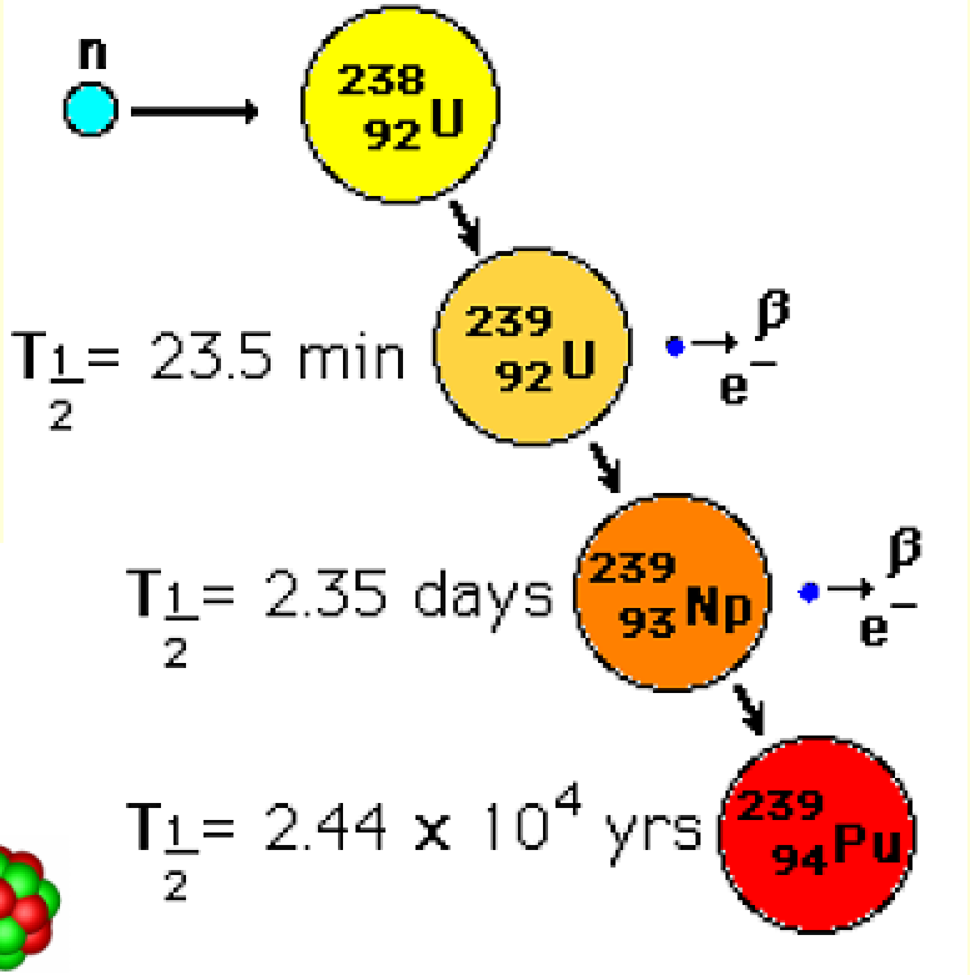
Energy per Fission

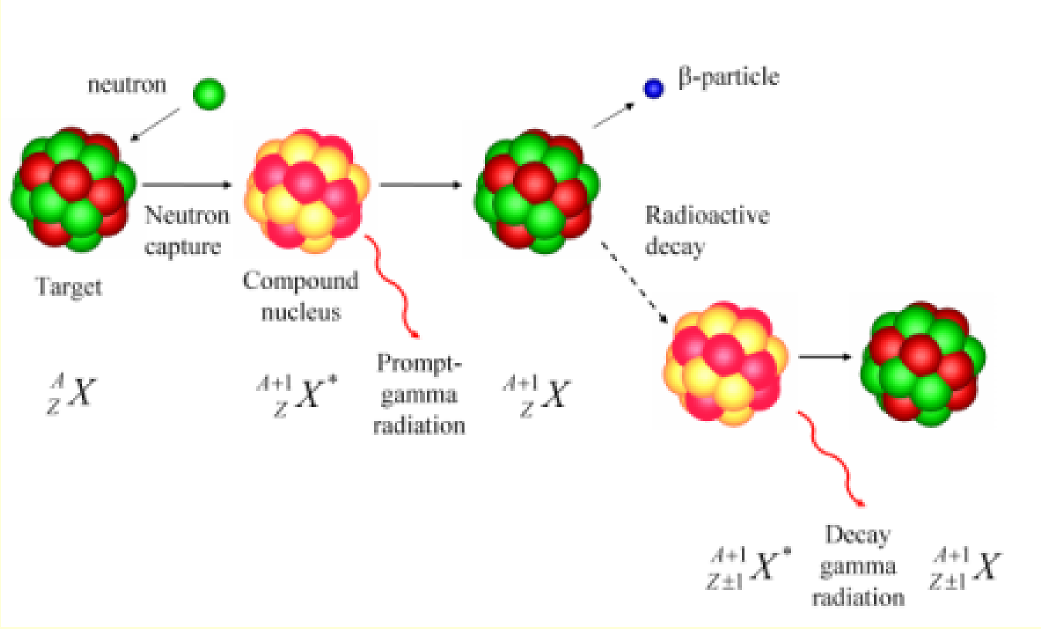
ENERGY DISTRIBUTION FOR FISSION INDUCED BY THERMAL NEUTRONS IN ^{235}U

Source	*Energy (MeV)*
Fission product kinetic energy	168
Neutron kinetic energy	5
Fission γ's (instantaneous)	5
Fission γ's (delayed)	6
Fission product β's	7
Total available as heat	191
Neutrino energy (not available as heat)	11
TOTAL	202

Neutron Capture: n,γ

- Addition of one neutron to a nuclide without fission
- Parasitic or productive

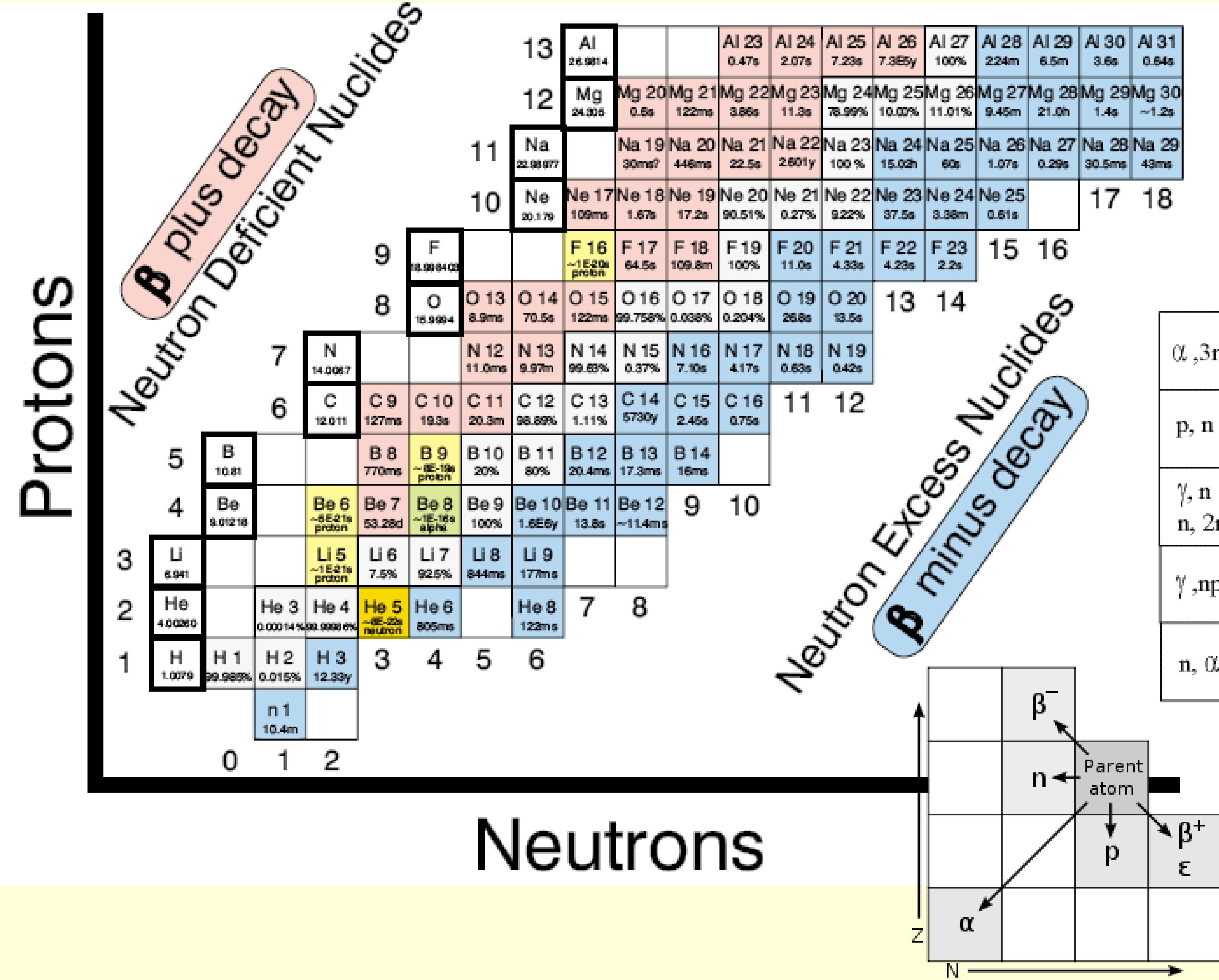


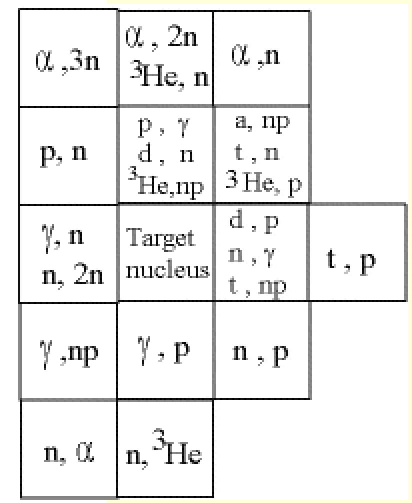


Neutron Capture: Other

- Neutron capture can lead to results other than gamma rays
  - Neutrons out: (n, 2n); (n, 3n)
  - Charged particles out: (n, p), (n,α)
  - Tend to require high-energy neutrons and have small cross sections
    - Exception: When capture product is magic
- Notation: Absorption cross section is the sum of all neutron capture and fission reactions

Nuclear Transformations





Neutron speed, energy, temperature

- $E(J) = 0.5 * 1.6755 \times 10^{-27}$ kg $* (2200$ m/s$)^2$

  $E(J) = 4.055 \times 10^{-21}$ J

- $E(eV) = 4.055 \times 10^{-21}$ J $* 6.24 \times 10^{18}$ eV/J

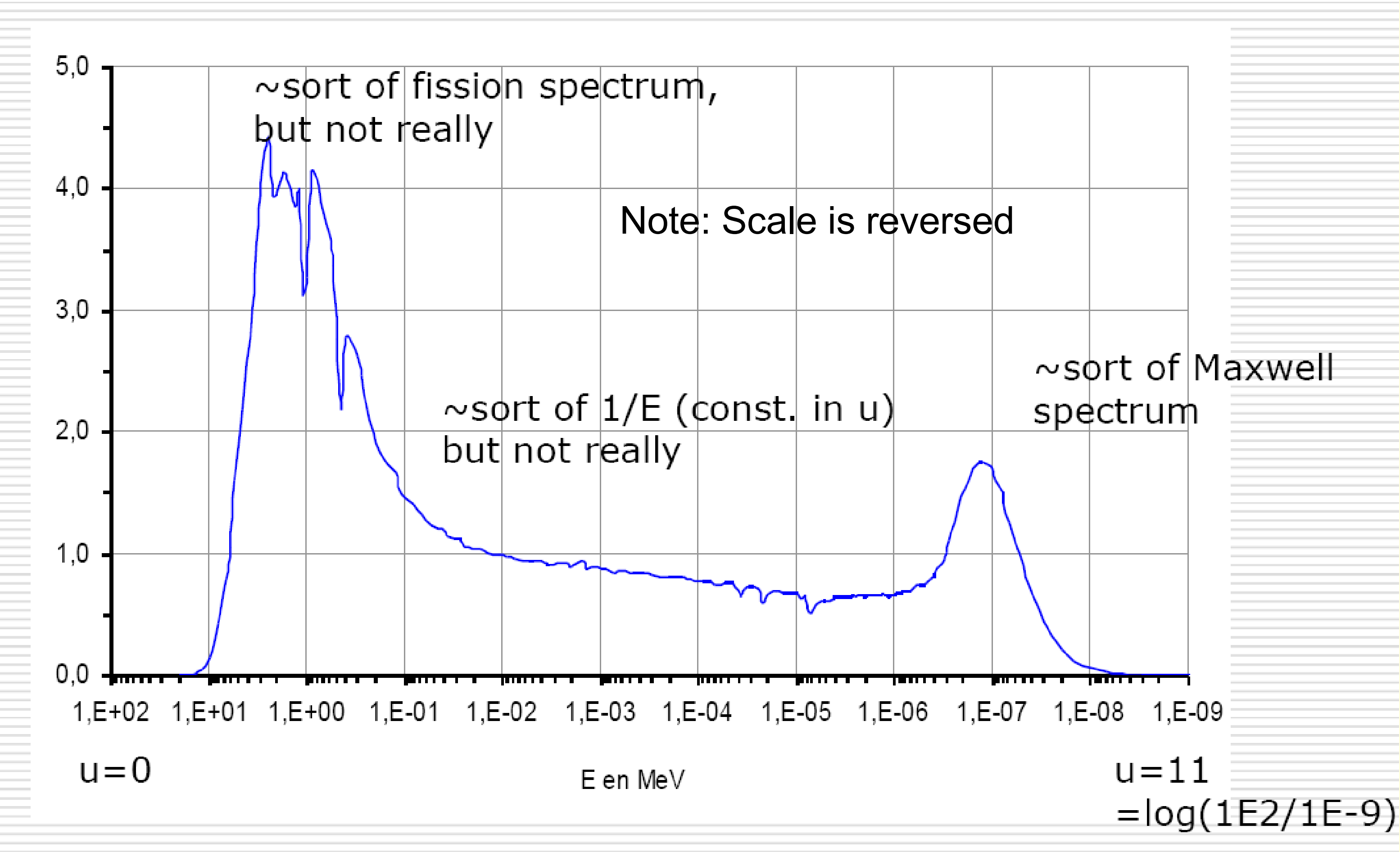
  $E(eV) = 0.0253$ eV

Boltzman Constant

- $E(T) = 4.055 \times 10^{-21}$ J / 1.38×10^{-23} J/K = 294 K
  - 294 K = 21 C = 70 F
  - Thermal neutron

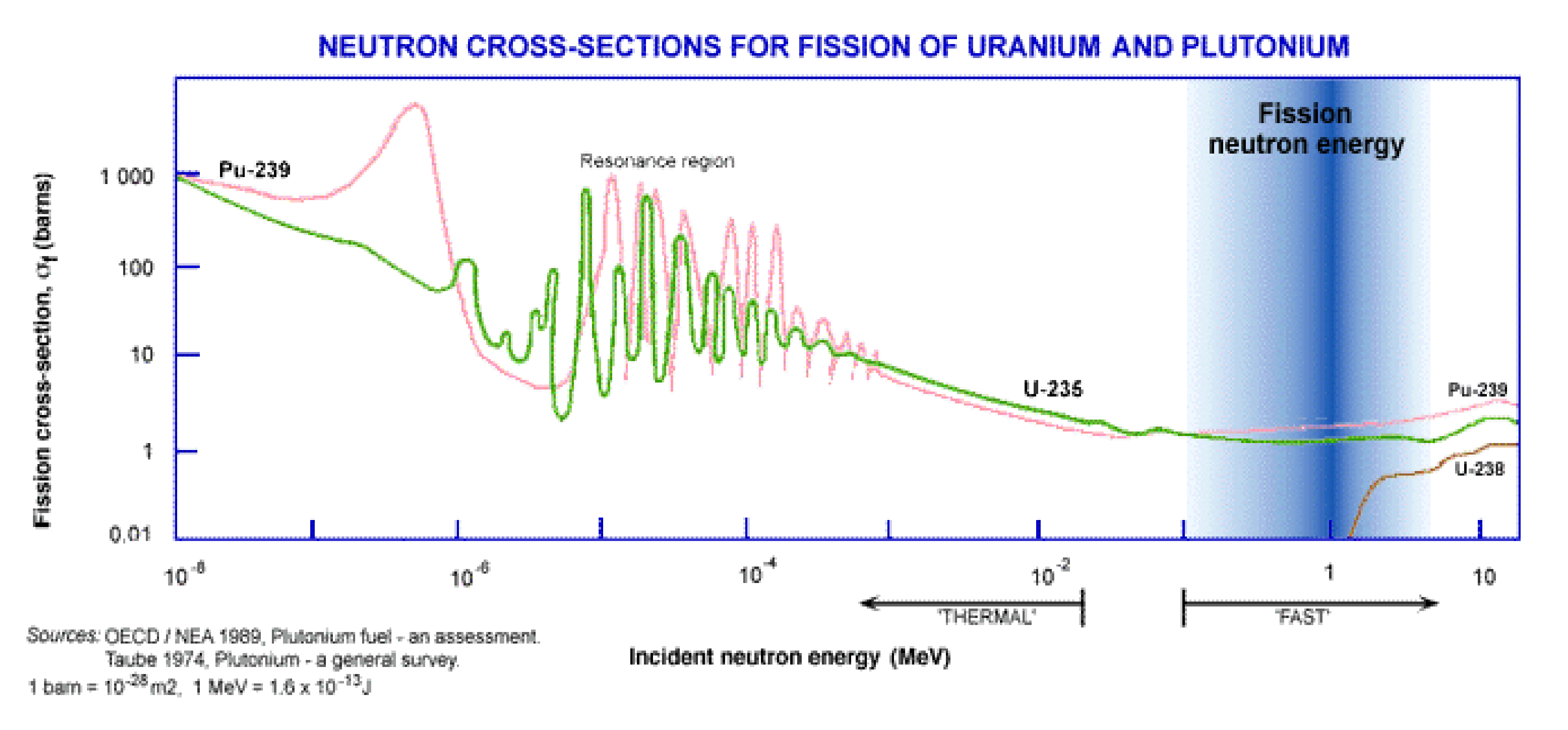
Range of Neutron Spectrum

- Neutron energy spectrum can range from fraction of an eV to ~15 MeV in a reactor



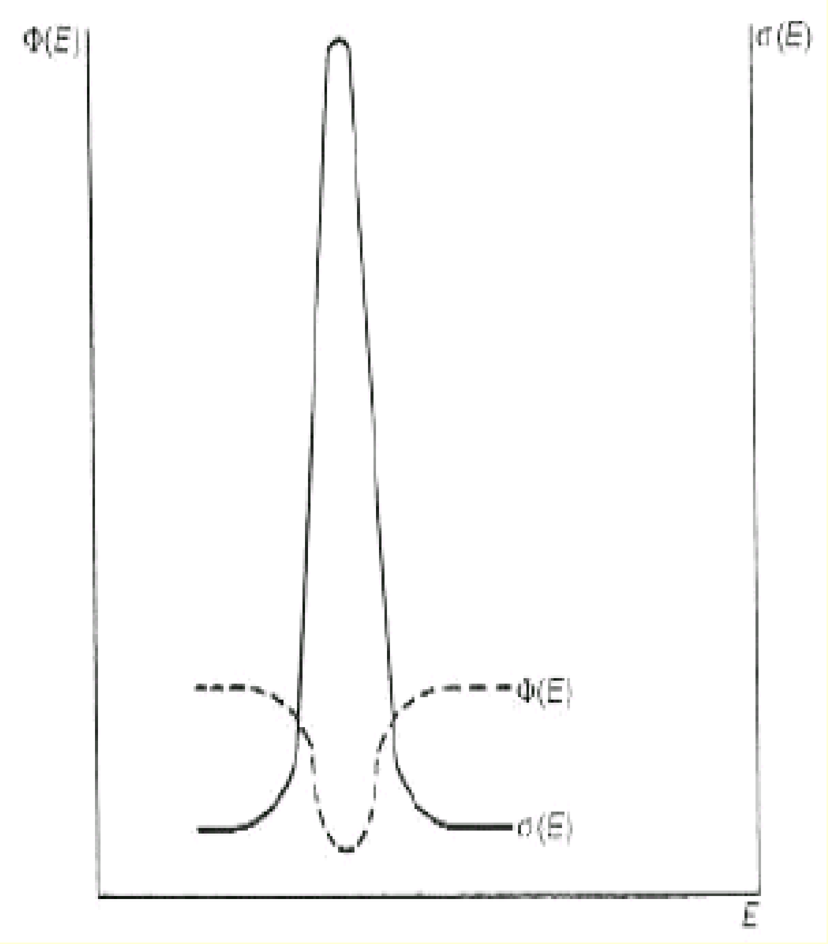
Typical Cross Sections

- Complication: Cross sections often vary by orders of magnitude across the neutron spectrum



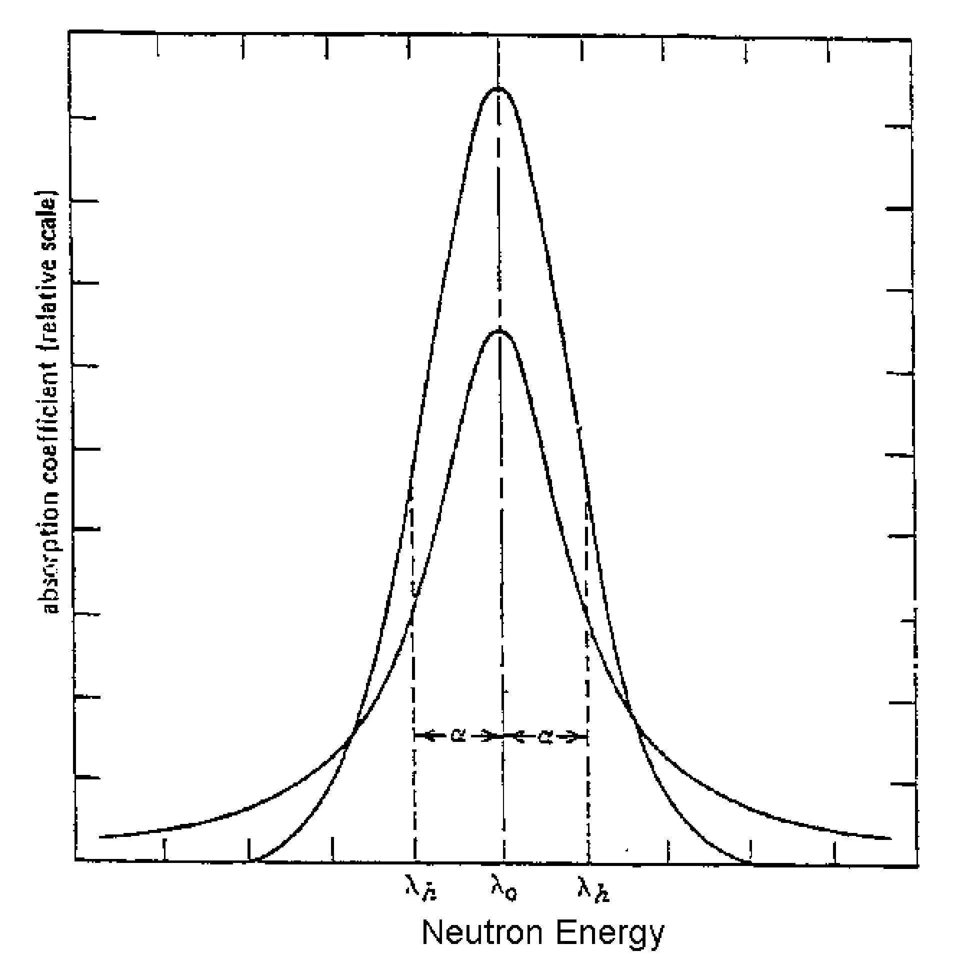
Effective Size of a Resonance

- Complication:
  Resonance self shielding
- Resonances deplete supply of neutrons at a particular energy
  - In effect, resonances are smaller than measured
  - Effect can occur because resonance is large and/or concentration of nuclide is high



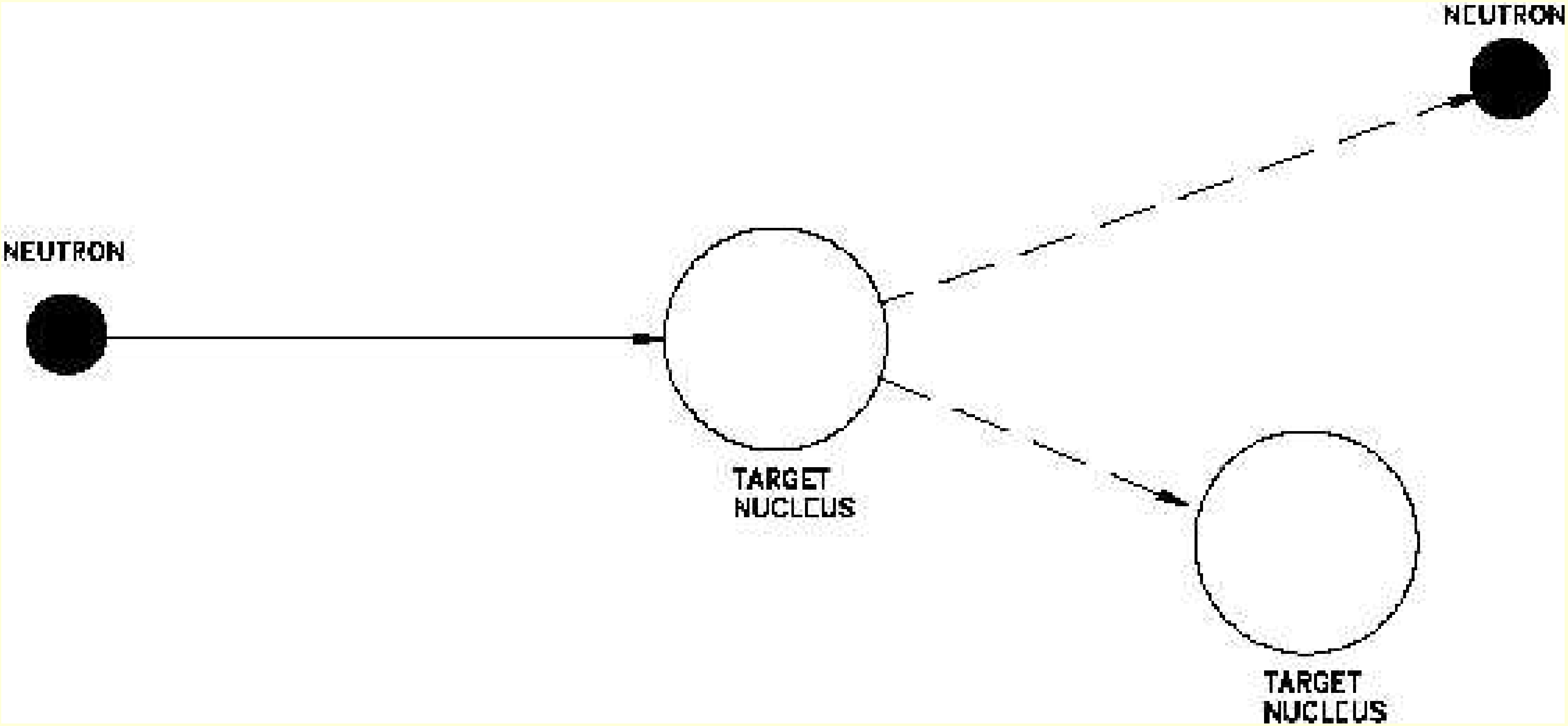
Doppler Broadening

- Resonance data measured at room temperature
- Complication: resonances get shorter and wider as temperature increases
  - Net effect is to increase size of the resonance



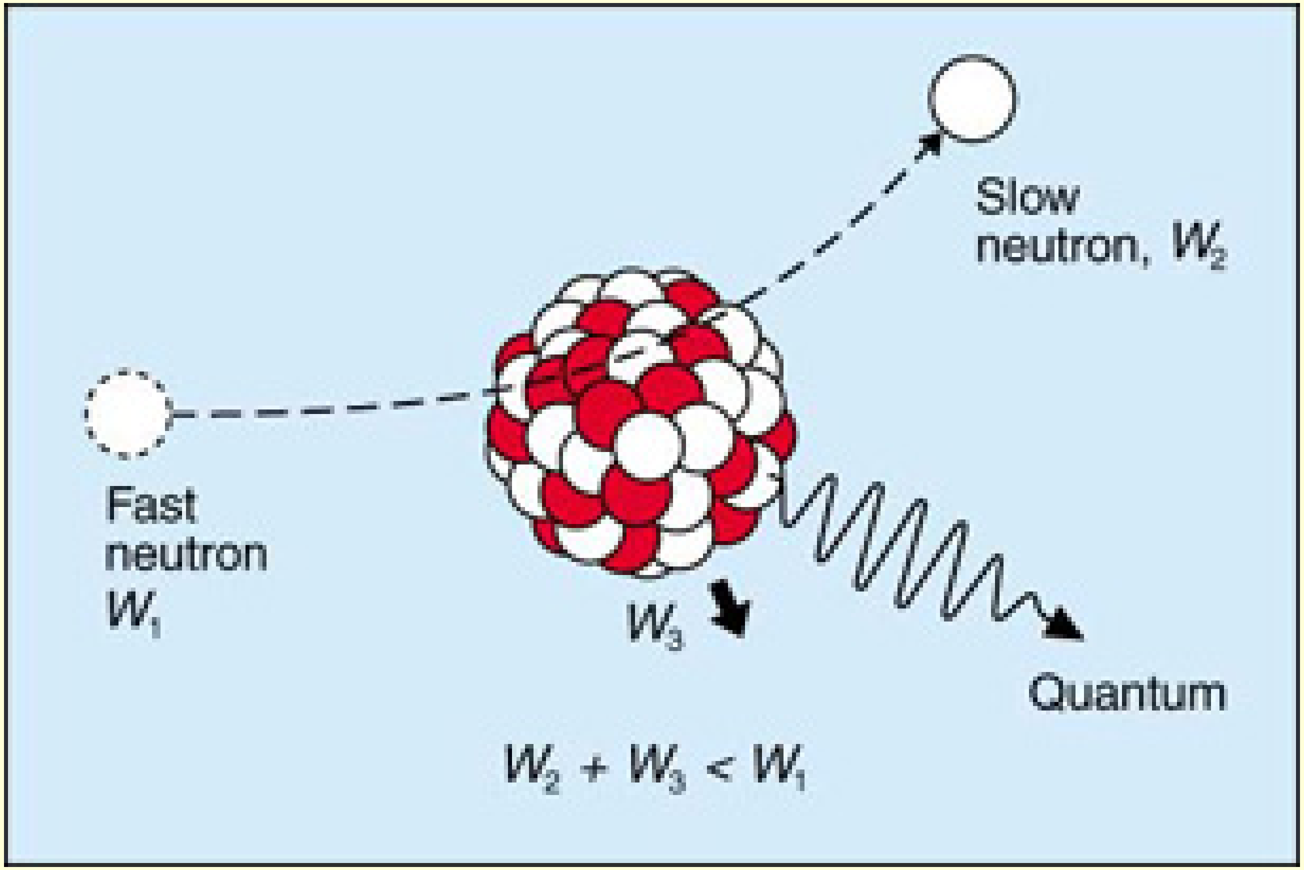
Neutron Scattering: Elastic

- Elastic neutron scattering: no change in total kinetic energy of particles
  - 'Billiard ball' model



Neutron Scattering: Inelastic

- Kinetic energy of final particles is less than that of initial particles
  - Energy difference released as gamma rays



Neutron Moderation

- Complication: For many reactors it is desirable for the cross section to be as large as possible
  - $R = N^*\sigma^*\ \varphi = \Sigma^*\ \varphi$
  - Reduces the required concentration of nuclear material in the fuel
- Need to slow neutrons to thermal energies
  - Notation: Neutron moderation
- Moderation occurs by neutron scattering off moderator nuclei

Neutron Moderation

- Criteria for a good neutron moderator
  - Maximizes neutron scattering
  - Minimizes non-productive neutron capture
  - Has a low atomic mass
    - Maximum neutron energy loss per collision is proportional to

$$1 - \left(\frac{A-1}{A+1}\right)^2$$

which is at a maximum when A=1 and drops to 0.28 for A=12

Neutron Moderation

- Moderating ratio takes criteria into account

Moderator	Moderating Ratio
Water	58
Heavy water	21,000
Helium, 1 atm	10^{-5}
Beryllium	130
Graphite	200

Thermal Reactor Criticality and Control

What is a thermal reactor

- A nuclear reactor in which the neutrons are moderated and most fissions are caused by thermal neutrons
  - Thermal neutrons = neutrons in thermal equilibrium (~Maxwelian distribution) at the temperature of their environment

Thermal Neutron Cycle

Step in Neutron Cycle	Factor	Change in number of neutrons	Total number of neutrons	Notation
Start: 1000 fission neutrons			1000	
Fast fission factor- mainly U-238	1.044	44.0	1044	ε
Fast neutron non-leakage probability	0.861	-145.1	899	P_1
Epithermal neutron non-leakage probability	0.952	-43.1	856	P_2
Epithermal neutrons escaping U-238 capture	0.816	-157.5	698	p_{28}
Epithermal neutrons escaping non-fuel capture	0.970	-20.9	677	p_c
Epithermal fissions in U-235	1.034	23.7	701	B
Thermal neutron non-leakage probability	0.985	-10.5	691	P_3
Thermal neutrons absorbed in fuel (utilization)	0.817	-126.4	564	f
Reduced absorp because of fuel temp	0.990	-5.6	559	T
Net fission neuts per neut absorbed	1.790	441.3	1000	η

Getting to Critical

- First order effects
  - Increase concentration of fissile material
  - Decrease parasitic neutron absorption
    - Select materials with low cross sections
    - Less material in the reactor
  - Reduce neutron leakage
    - Neutrons leak from reactor surface
    - Larger reactors have lower surface-to-volume ratio

Reactor Power

- Power (watts) = $8.3 \times 10^{10} * \sigma_f * M * \varphi$

where σ_f = Fission cross section, cm^2

M = Mass of fissile material, g

φ = Neutron flux, neutrons/cm^2-sec

Reactor Control

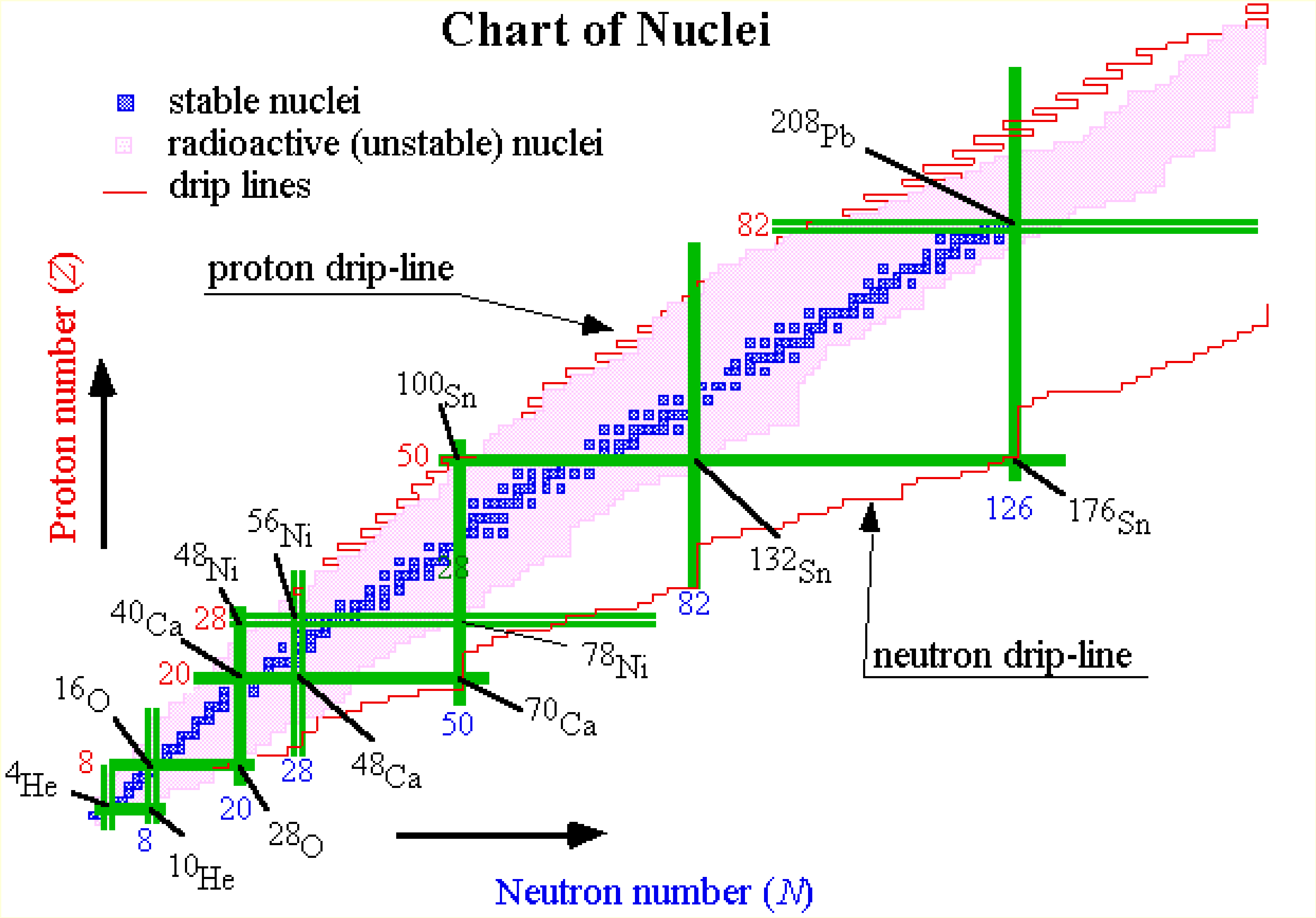
Prompt Neutrons

- On average, each fission releases 2.5 to 3 fast neutrons within 10^{-13} seconds
- The time from one generation of prompt neutrons to the next is ~10^{-5} seconds
- Stable period: the time it takes to increase the neutron flux (and reactor power) by e (2.72x)
- For prompt neutrons the stable period is a fraction of a second
- Under these conditions a reactor would be unstable and uncontrollable

Delayed Neutrons

- A small fraction of fission products decay by emitting neutrons
- These neutrons are emitted at lower energy than prompt neutrons and with a defined half-life along the neutron drip line
- The existence of decay neutrons increases the stable period to several seconds which allows the reactor to be controlled

Neutron Drip Line



Nuclear Reactor Control

- Neutron poison: A non-productive neutron absorbing material typically having a large neutron capture cross section
  - More accurately “fission poison”
- Approach: Vary the amount of neutron absorber (i.e., the amount of non-productive neutron absorption) to achieve stable operation or slowly changing power levels

Neutron Poisons

Material	Macroscopic Thermal Absorption Cross Section, barns
Boron	107
Silver	4
Cadmium	113
Indium	7
Samarium	155
Europium	90
Gadolinium	1400

Inherent Control Mechanisms

- Increased temperature normally decreases neutron interaction
  - Doppler broadening: more neutrons absorbed in resonances
  - Thermal expansion of fuel and core: more neutron leakage
  - Boiling of coolant: less moderation
    - This effect can increase neutron interactions in fast reactors under some circumstances

Reactor Physics Calculations

Definition

- The science of the interaction of elementary particles and radiations characteristic of nuclear reactors with matter in bulk

What Needs to be Known?

- Neutron flux in and around a reactor core, and interactions and reactions of neutrons and other radiation in fuel and structural materials as a function of
  - Space
  - Energy
  - Direction
  - Time

How It is Done

- Solve the Boltzman equation describing transport of neutral particles
  - Essentially the same approach for neutrons and photons

Boltzman Equation

$$\frac{1}{v(E)}\frac{d\Phi(\mathbf{r}, E, \mathbf{\Omega}, t)^{①}}{dt}$$

$$= -\mathbf{\Omega}\cdot\nabla\Phi(\mathbf{r}, E, \mathbf{\Omega}, t)^{②} - \Sigma_t(\mathbf{r}, E, \mathbf{\Omega})\Phi(\mathbf{r}, E, \mathbf{\Omega}, t)^{③}$$

$$+ \chi(E)\int_{E'} dE' \int_{\Omega'} d\Omega'\ \nu\Sigma_f(\mathbf{r}, E', \mathbf{\Omega}')\Phi(\mathbf{r}, E', \mathbf{\Omega}', t)^{④}$$

$$+ \int_{E'} dE' \int_{\Omega'} d\Omega'\ \Sigma_s(\mathbf{r}; E' \rightarrow E; \mathbf{\Omega}' \rightarrow \mathbf{\Omega})\Phi(\mathbf{r}, E', \mathbf{\Omega}', t)$$

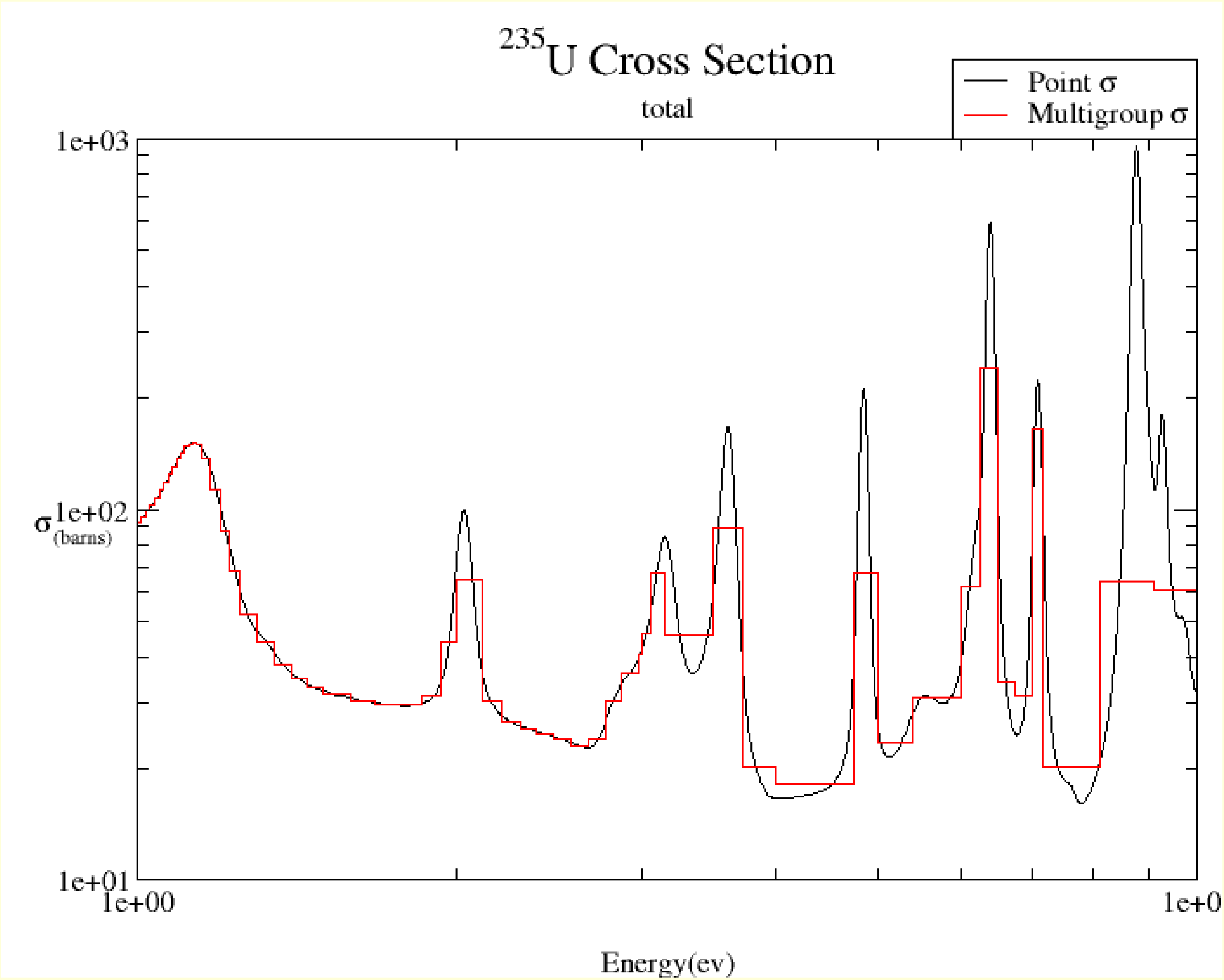
Term 1 = rate of accumulation of neutrons
Term 2 = rate of leakage out of the element
Term 3 = total interaction rate (removal due to absorption or scatters out of volume element or out of energy, E).

Terms 4 and 5 represent production phenomena where neutrons at E' and Ω' react with nuclei to generate neutrons at E and Ω. Integrals sum over all initial energies and direction.

Term 4 = total fission rate; then χ(E) refers to the energy spectrum produced in the fission process.

Term 5 = differential scattering of neutrons from initial energy, E', to final energy, E, and from initial direction Ω', to final direction, Ω. The cross section, E_s(r;E'→E; Ω'→Ω) accounts for the relative probabilities of all possible combinations.

Step 1: Multigroup Approximation

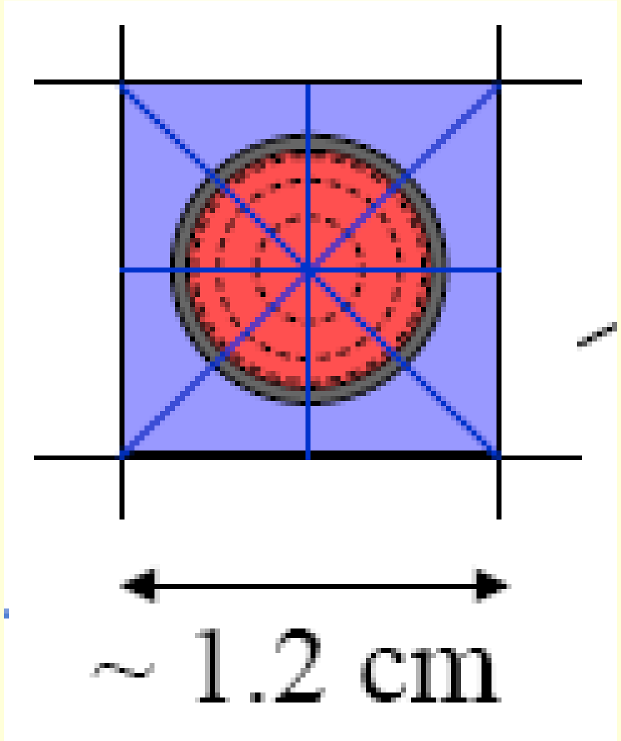


Boltzman Solution Complication

- Accounting for heterogeneity of reactors: with variation in space, energy, neutron angle, time the number of points is in the billions

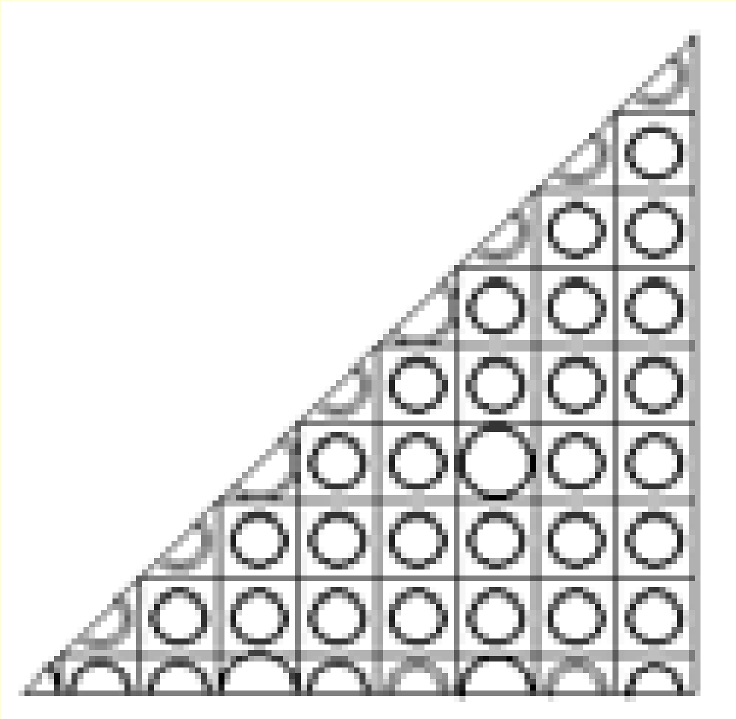
Step 1: Start Small with Detail

- Start with a single fuel rod
- Using transport or Monte Carlo methods and multigroup cross sections calculate the many group neutron flux
  - Static
  - Two dimensions
  - Only key nuclides, lumped fission products
- Done for a number of fuel compositions



Step 2: Start Small with Detail

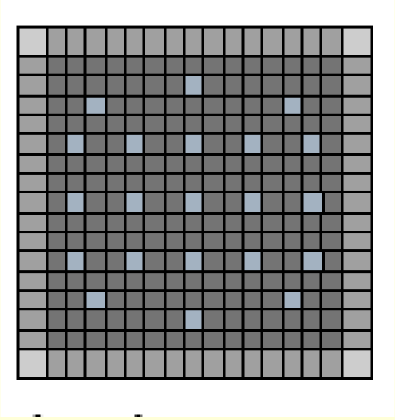
- With modern computing power it may be possible to do many-group modeling of a number of symmetrical cells in an array



Step 3: Homogenize and Grow

- Use the many group flux to weight the many group cross sections to yield few (2 to 5) group cross sections
- Reaction rates (product of flux and cross section are preserved) during homogenization

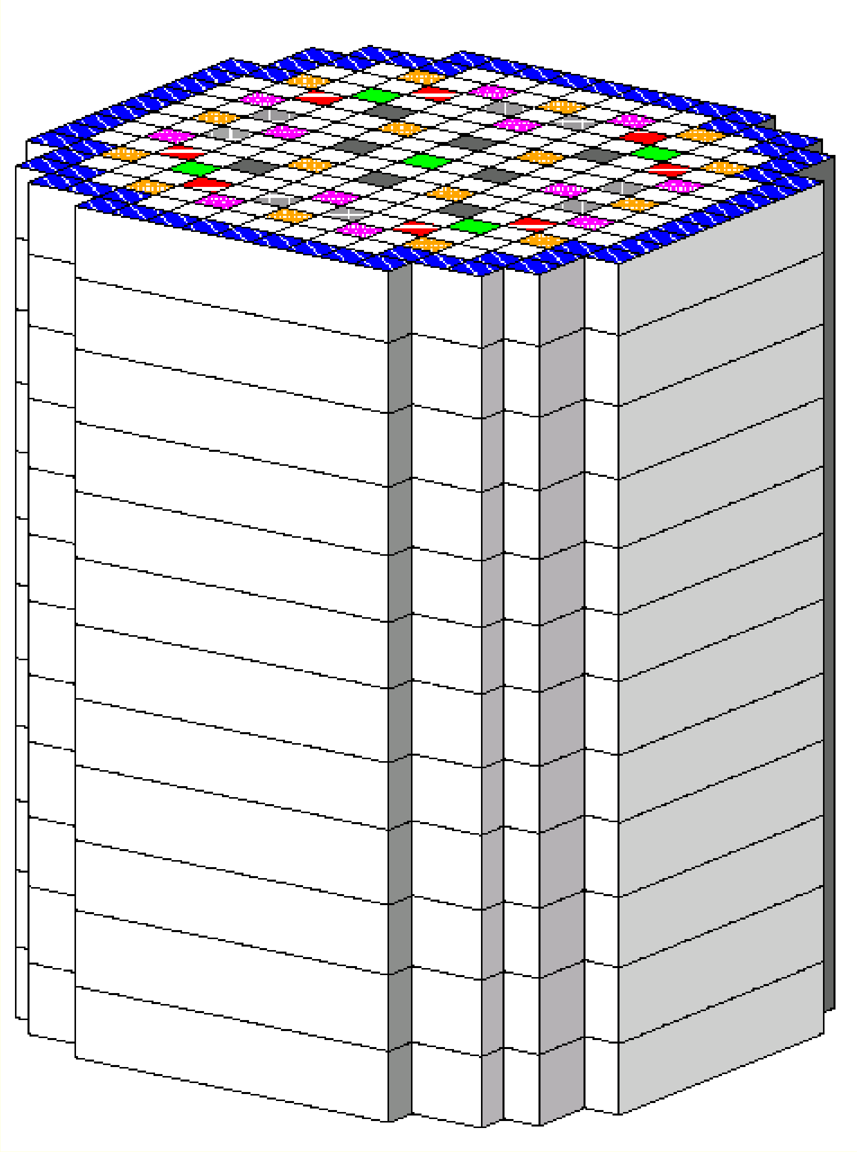
Array



Cells

Step 4: The Whole Core

- 3-D model of entire core
  - Time dependent
  - Few energy groups
  - Relatively coarse grid
- Stepwise iteration with depletion (next slide)
- Diffusion theory is fast and sufficiently accurate when homogenized
- 3-D codes are run many times to optimize fuel composition and movement



Step 5: Detailed Depletion

- Use few group fluxes to weight cross sections for many nuclides (hundreds) to yield total flux and spectrum-averaged (1-group) cross sections
- Use cross sections and flux in models to predict the buildup and decay of many radionuclides as a function of time
  – Simple conversions yield other properties such as gamma ray intensity and decay heat
- Boltzman equation not needed: solve the Bateman equations

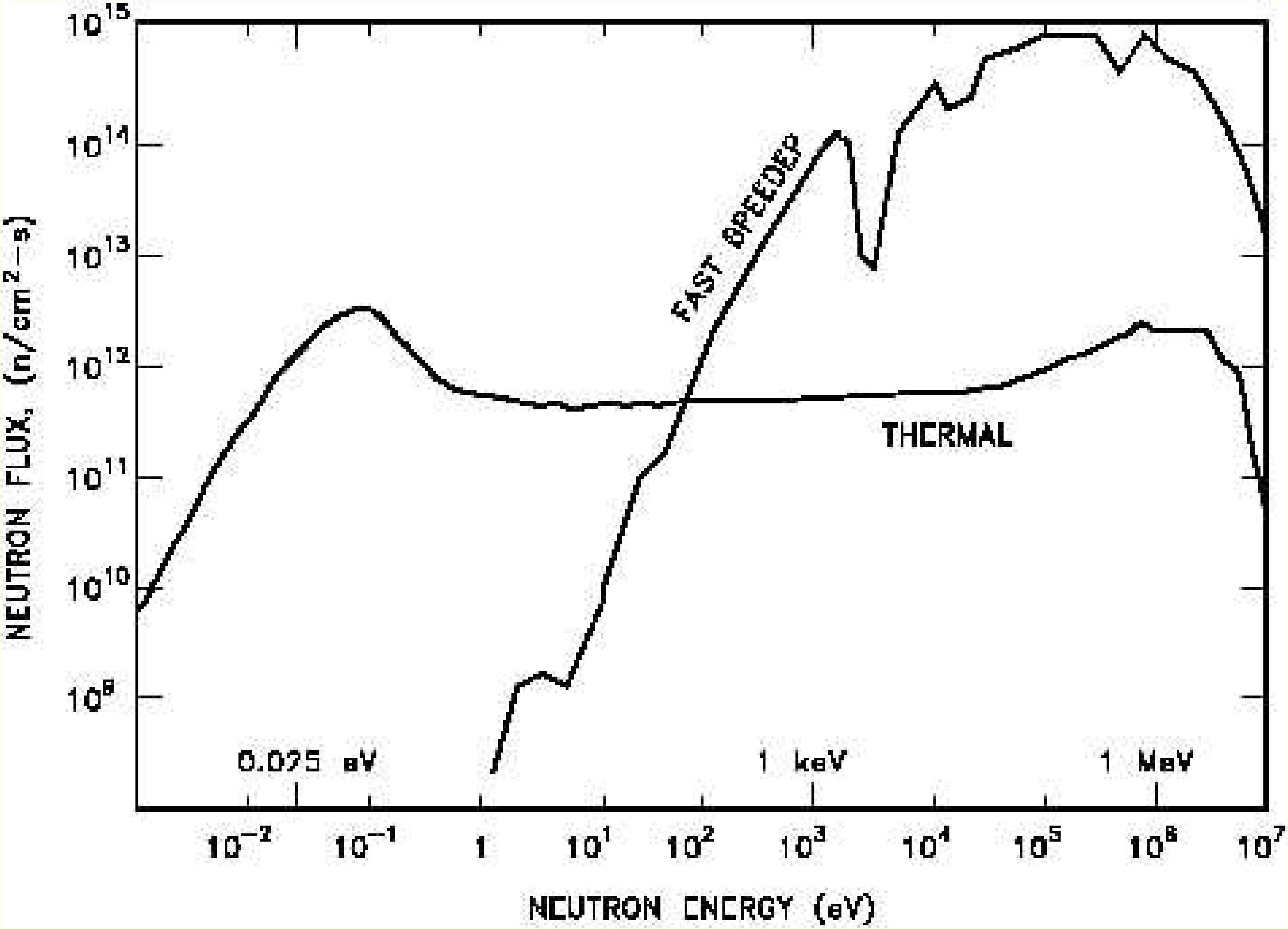
Bateman Equation

- The Bateman equation describes the buildup and decay of nuclides given knowledge of flux and cross sections
  - It can be solved in closed form for many cases
    - Closed form is tedious but computer codes alleviate this
    - Complication: Closed form not possible when a radionuclide produces itself
      - Common for the actinides because of alpha decay
    - Solution: Solve equation numerically
    - Computer codes exist; results later

$$\frac{dN_i}{dt} = -\sum_{j\neq i}\left[\lambda_{ji}^{d} + \int \varphi(E,t)\sigma_{ji}^{tr}(E)dE\right]N_i + \sum_{j\neq i}\left[\lambda_{ij}^{d} + \int \varphi(E,t)\sigma_{ij}^{tr}(E)dE\right]N_j$$

Fast Reactor Physics

Fast Reactor Neutron Spectrum



Ramifications of Fast Spectrum-1

- Criticality considerations simpler
  - No need to consider thermal region
  - Resonances are much less important
- But
  - Fissions in fissionable nuclides (e.g, U^{238}) are more important

Ramifications of Fast Spectrum-2

- Cross sections are smaller at higher energy because of general 1/v dependence
  - Need higher concentration of fissile material to achieve criticality
  - Neutron fluxes are higher to achieve desired power level

Ramifications of Fast Spectrum-3

- Higher ratio of fission-to-absorption cross sections
  - Fewer neutron losses to unproductive capture
  - Convert fertile nuclides to fissile nuclides faster than they consume fissile nuclides: they can be “breeders”
  - Less production of minor actinides: Np, Am, Cm